上海大学未来技术学院 | 上海大学人工智能研究院
SCHOOL OF FUTURE TECHNOLOGY, SHANGHAI UNIVERSITY | INSTITUTE OF ARTIFICIAL INTELLIGENCE, SHANGHAI UNIVERSITY

人工智能导论

——第2课：人工智能基本理论

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未来技术学院（人工智能研究院）

2023冬季学期

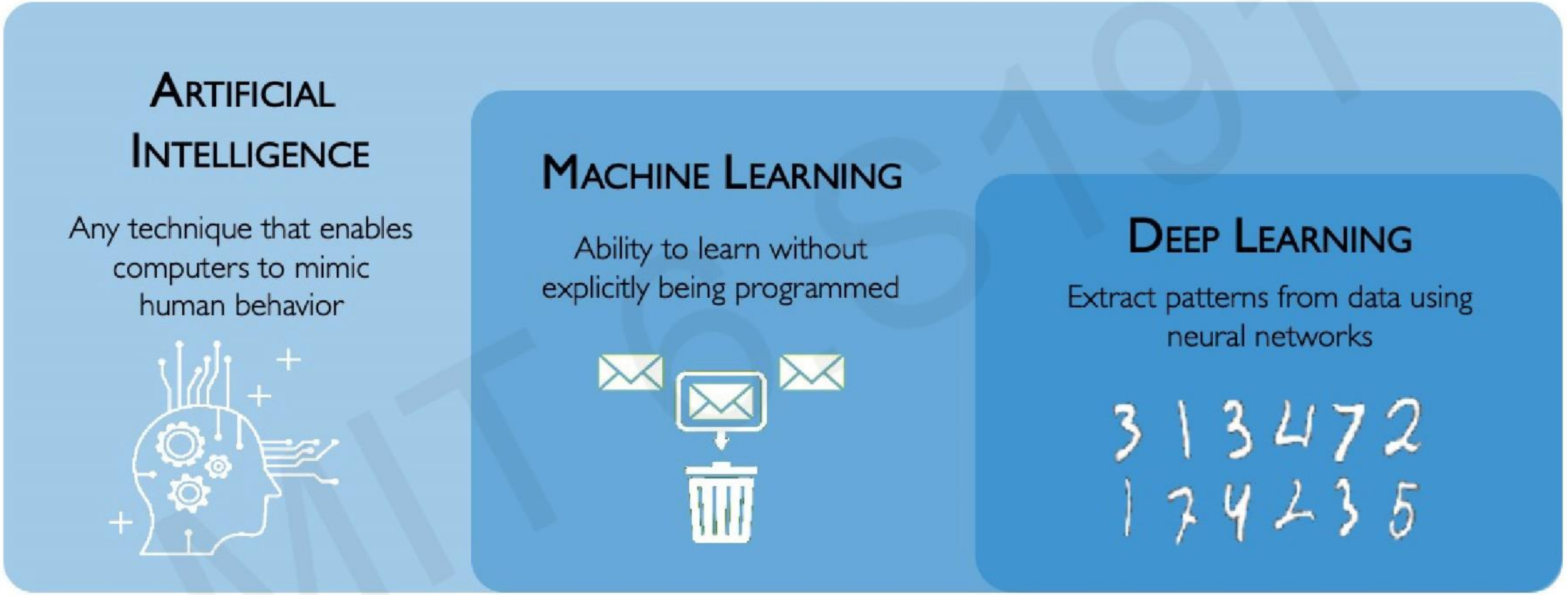
提纲

一、前馈网络

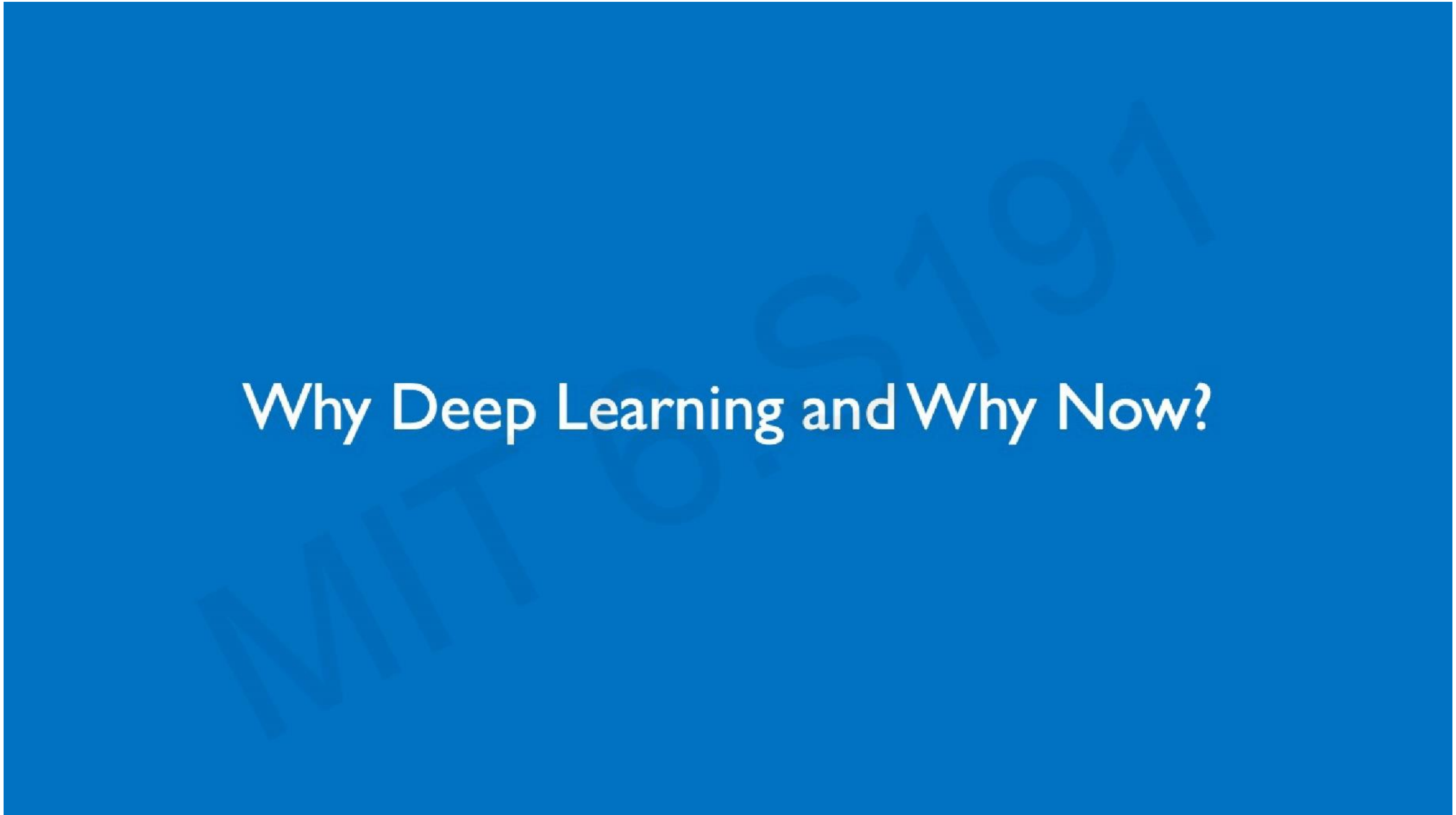
二、循环网络

三、卷积网络

What is Deep Learning?



Teaching computers how to **learn a task** directly from **raw data**

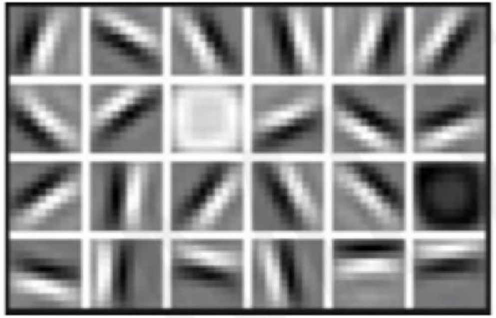

Why Deep Learning and Why Now?
MIT 6.S191

Why Deep Learning?

Hand engineered features are time consuming, brittle, and not scalable in practice

Can we learn the **underlying features** directly from data?

Low Level Features



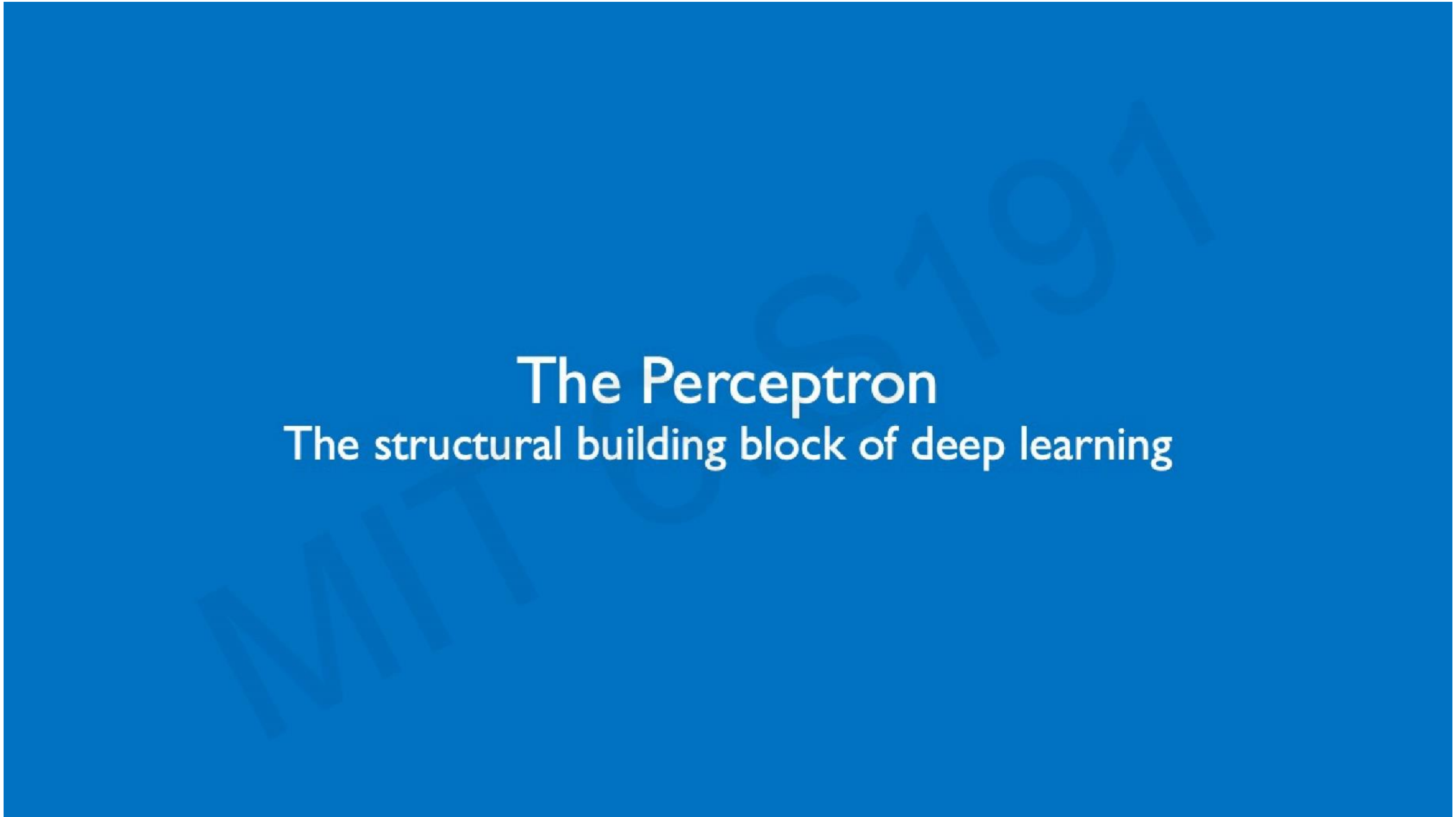
Lines & Edges

Mid Level Features

Eyes & Nose & Ears

High Level Features

Facial Structure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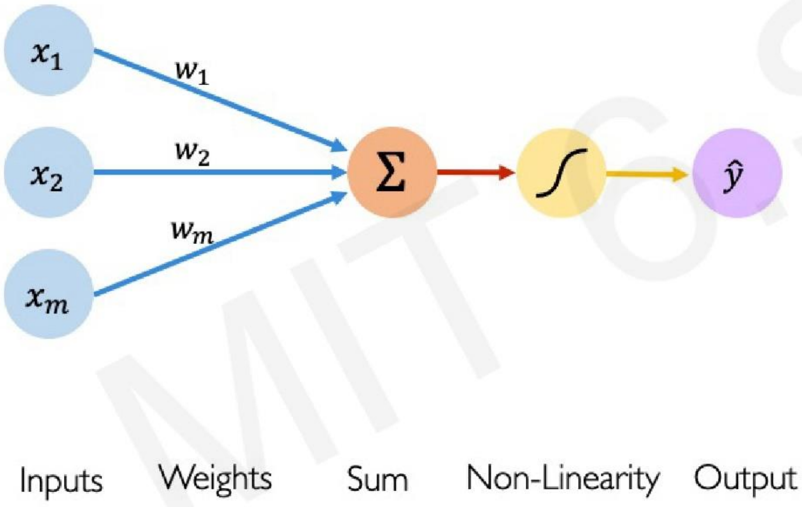
The Perceptron
The structural building block of deep learning

The Perceptron: Forward Propagation



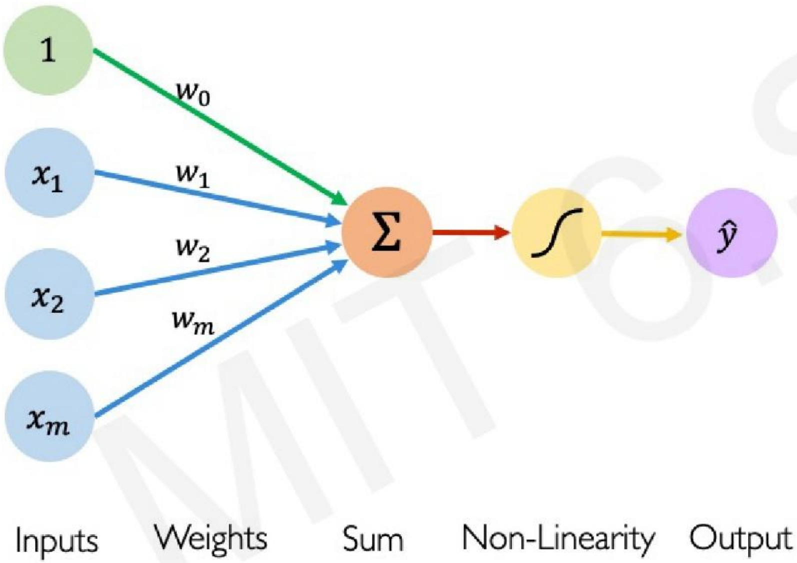
Output

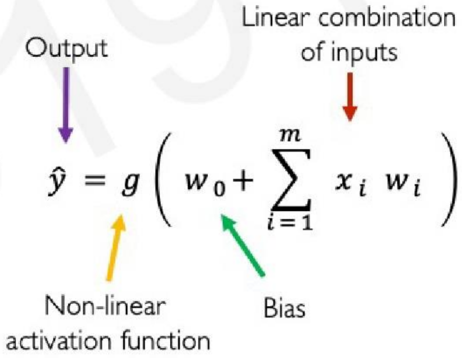
Linear combination of inputs

$$\hat{y} = g\left(\sum_{i=1}^{m} x_i \, w_i\right)$$

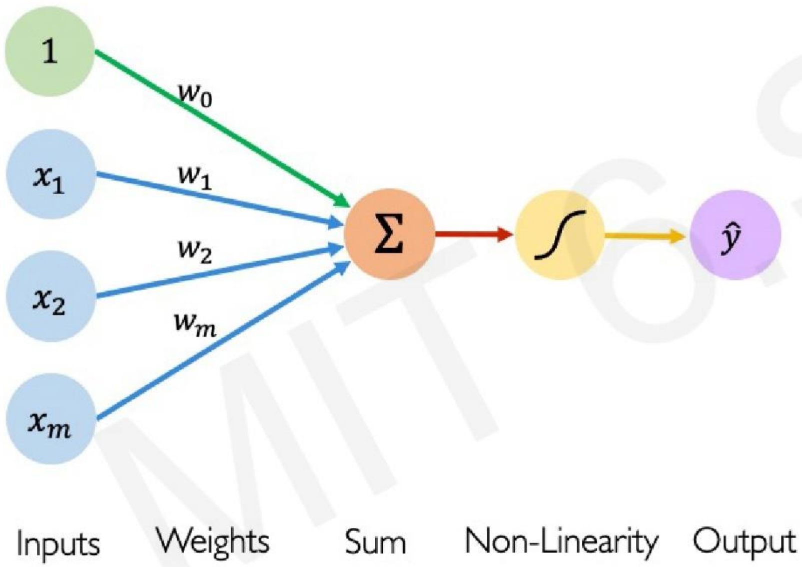
Non-linear activation function

The Perceptron: Forward Propagation





The Perceptron: Forward Propagation

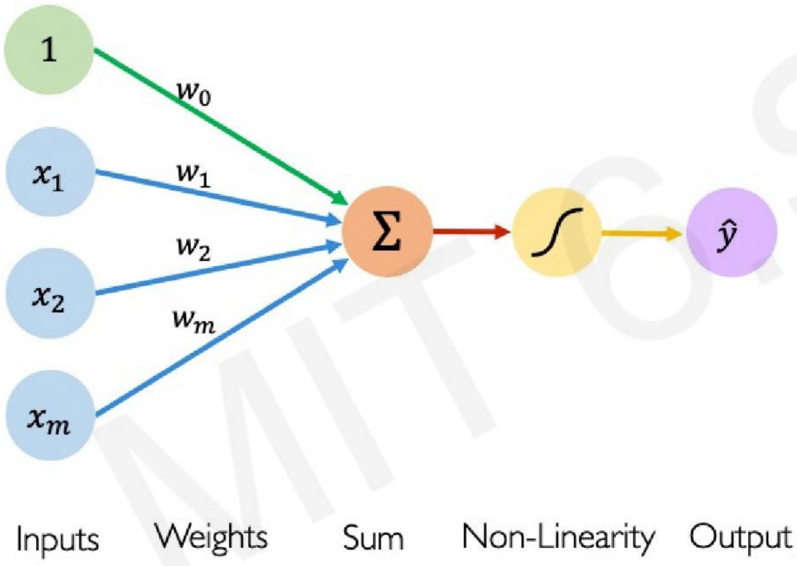


$$\hat{y} = g\left(w_0 + \sum_{i=1}^{m} x_i \, w_i \right)$$

$$\hat{y} = g\left(w_0 + \boldsymbol{X}^T \boldsymbol{W} \right)$$

where: $\boldsymbol{X} = \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix}$ and $\boldsymbol{W} = \begin{bmatrix} w_1 \\ \vdots \\ w_m \end{bmatrix}$

The Perceptron: Forward Propagation

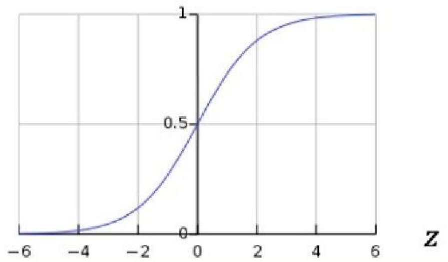


Activation Functions

$$\hat{y} = g\left(w_0 + \boldsymbol{X}^T \boldsymbol{W} \right)$$

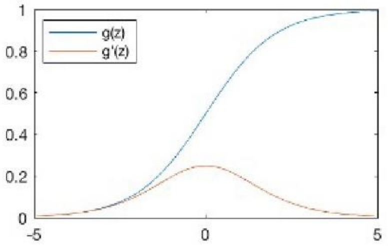
- Example: sigmoid function

$$g(z) = \sigma(z) = \frac{1}{1 + e^{-z}}$$



Common Activation Functions

Sigmoid Function

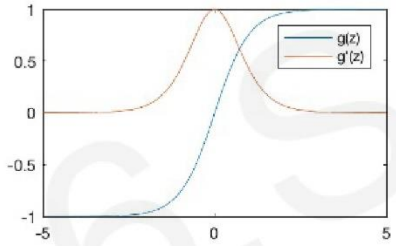


$$g(z) = \frac{1}{1 + e^{-z}}$$

$$g'(z) = g(z)(1 - g(z))$$

```
tf.math.sigmoid(z)
```

Hyperbolic Tangent

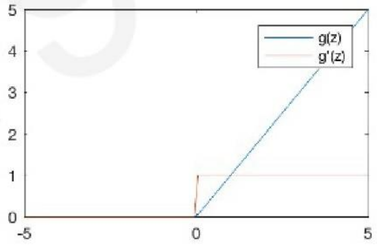


$$g(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$$

$$g'(z) = 1 - g(z)^2$$

```
tf.math.tanh(z)
```

Rectified Linear Unit (ReLU)



$$g(z) = \max(0, z)$$

$$g'(z) = \begin{cases} 1, & z > 0 \\ 0, & \text{otherwise} \end{cases}$$

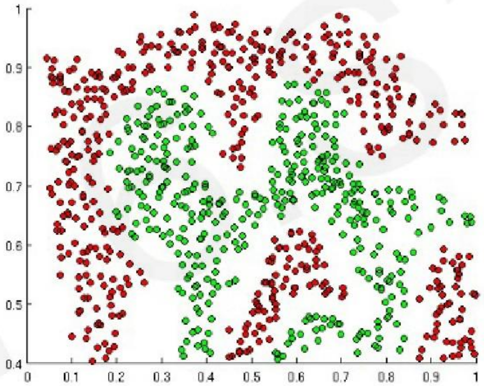
```
tf.nn.relu(z)
```

TensorFlow code blocks

NOTE: All activation functions are non-linear

Importance of Activation Functions

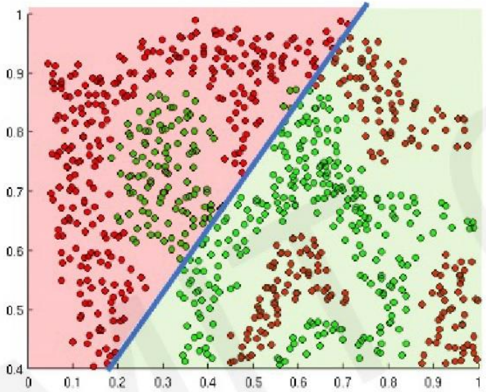
The purpose of activation functions is to ***introduce non-linearities*** *into the network*



What if we wanted to build a neural network to distinguish green vs red points?

Importance of Activation Functions

The purpose of activation functions is to ***introduce non-linearities*** *into the network*



Linear activation functions produce linear decisions no matter the network size

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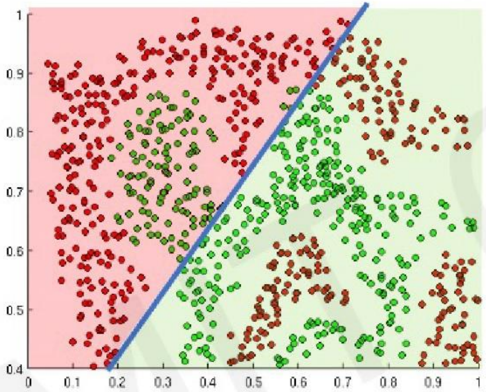
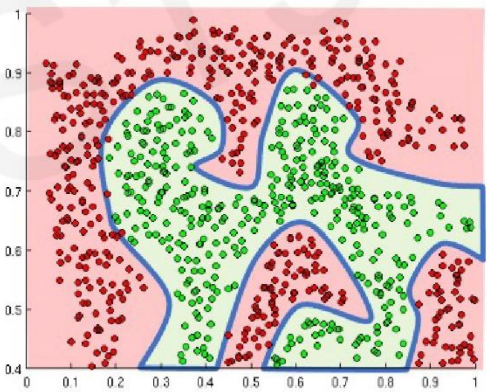
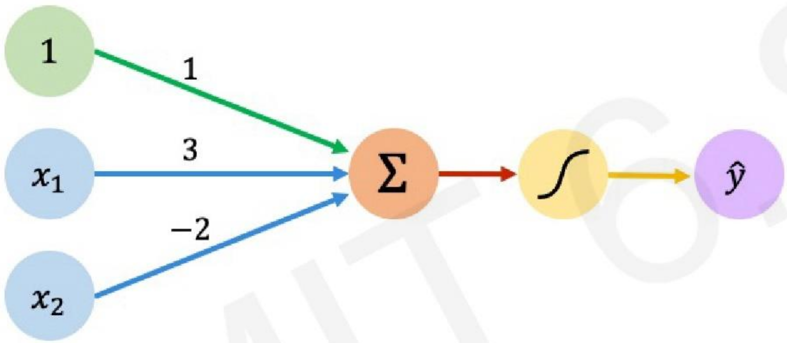


Linear activation functions produce linear decisions no matter the network size



Non-linearities allow us to approximate arbitrarily complex functions

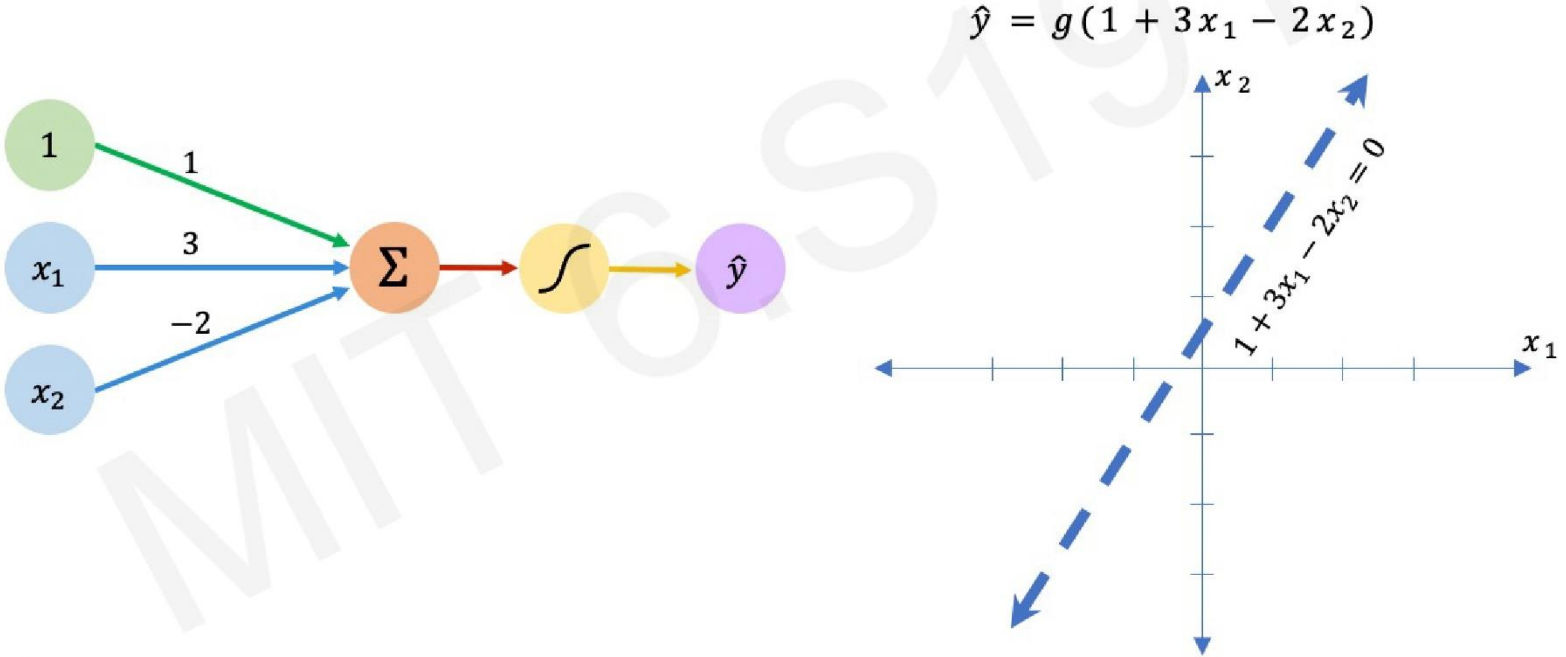
The Perceptron: Example



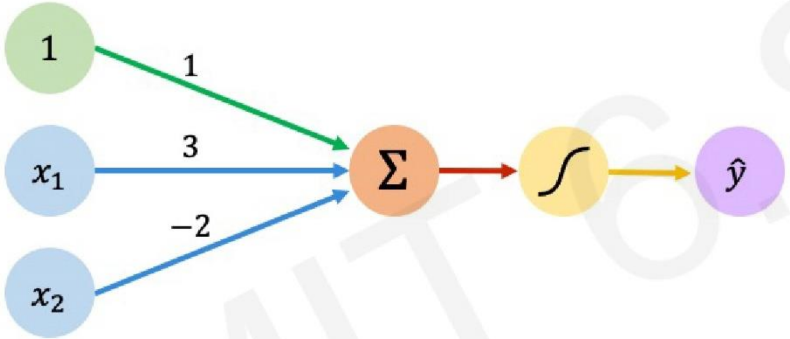
We have: $w_0 = 1$ and $\boldsymbol{W} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

$$\hat{y} = g\left(w_0 + \boldsymbol{X}^T\boldsymbol{W}\right)$$
$$= g\left(1 + \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} 3 \\ -2 \end{bmatrix}\right)$$
$$\hat{y} = g(\underbrace{1 + 3x_1 - 2x_2}_{\text{This is just a line in 2D!}})$$

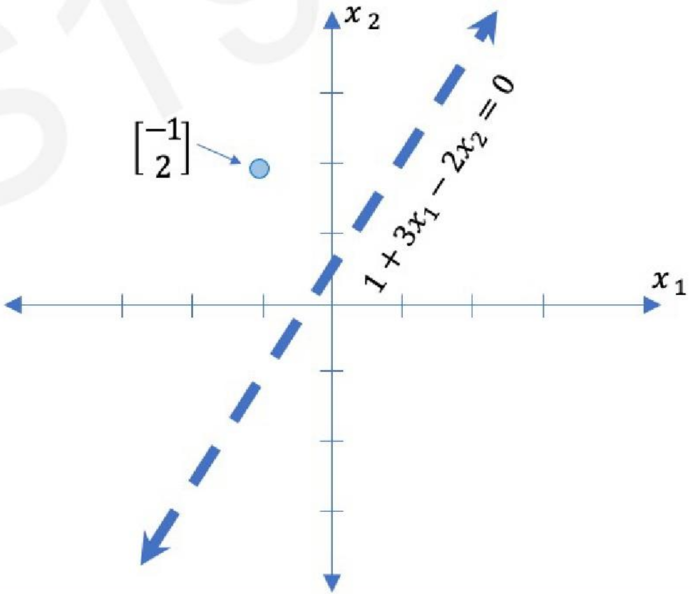
The Perceptron: Example



The Perceptron: Example



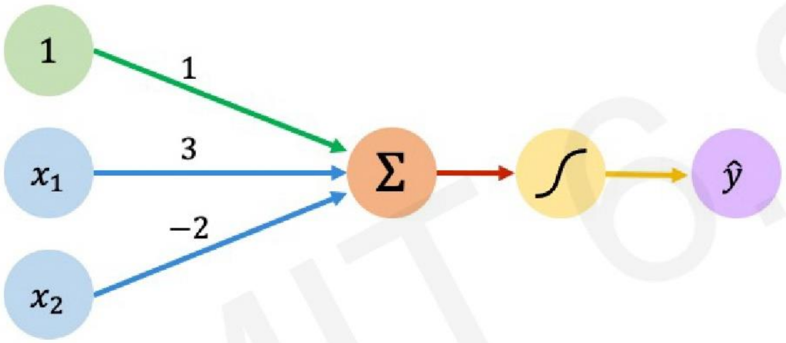
$$\hat{y} = g(1 + 3x_1 - 2x_2)$$

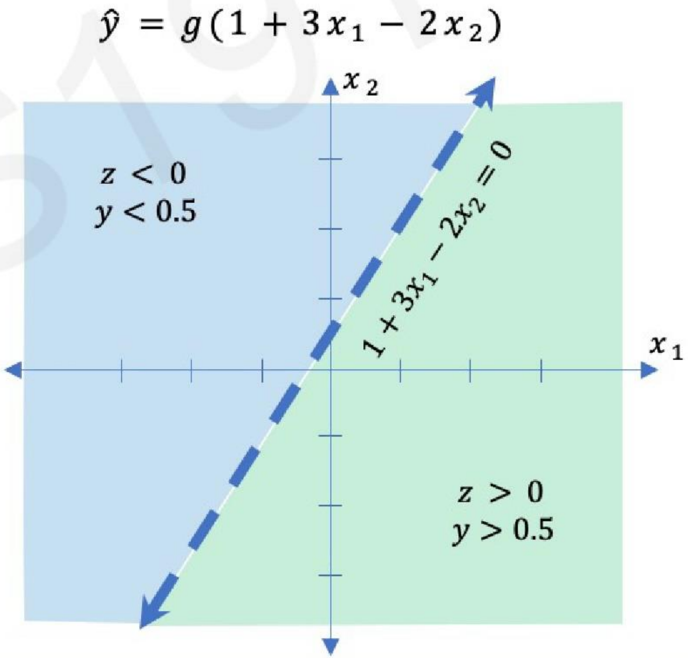


Assume we have input: $\boldsymbol{X} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$$\begin{aligned} \hat{y} &= g\big(1 + (3 * -1) - (2 * 2)\big) \\ &= g(-6) \approx 0.002 \end{aligned}$$

The Perceptron: Example

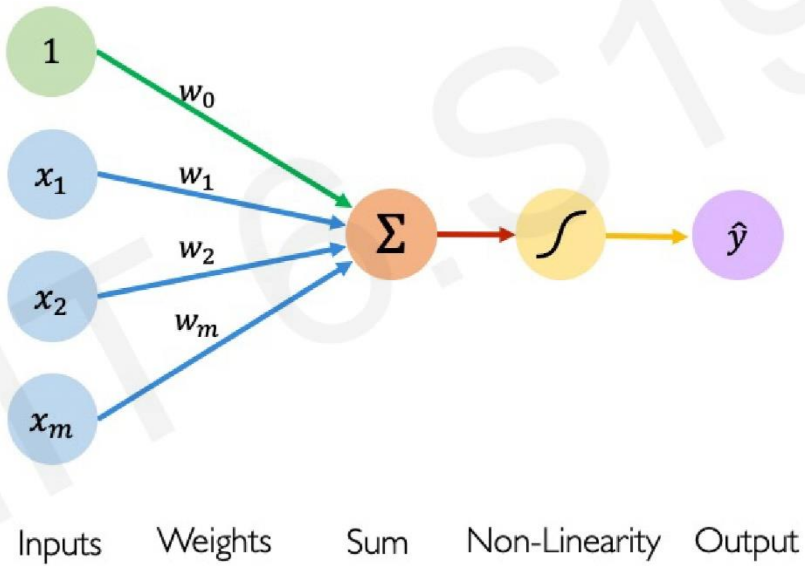




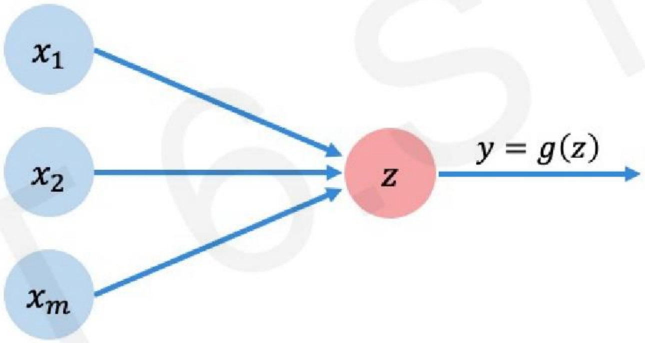
Building Neural Networks with Perceptrons

The Perceptron: Simplified

$$\hat{y} = g\left(w_0 + \boldsymbol{X}^T \boldsymbol{W} \right)$$



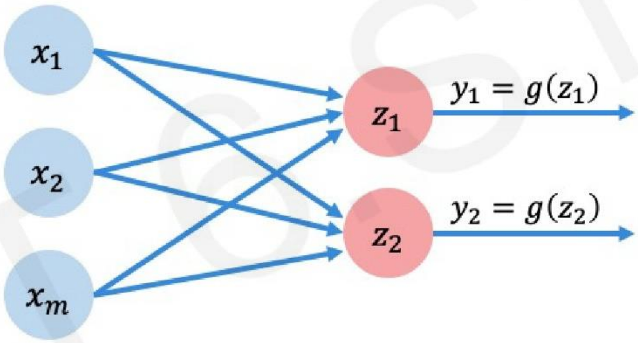
The Perceptron: Simplified



$$z = w_0 + \sum_{j=1}^{m} x_j w_j$$

Multi Output Perceptron

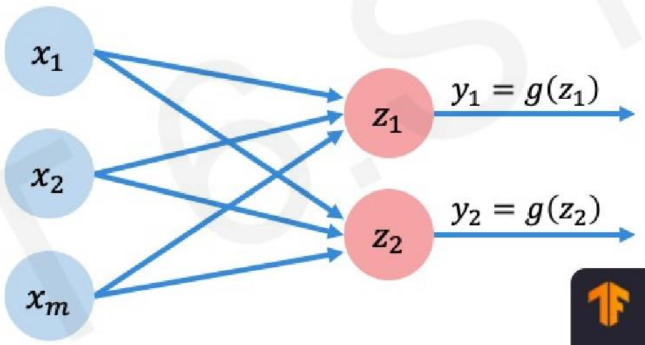
Because all inputs are densely connected to all outputs, these layers are called **Dense** layers



$$z_i = w_{0,i} + \sum_{j=1}^{m} x_j \, w_{j,i}$$

Multi Output Perceptron

Because all inputs are densely connected to all outputs, these layers are called **Dense** layers

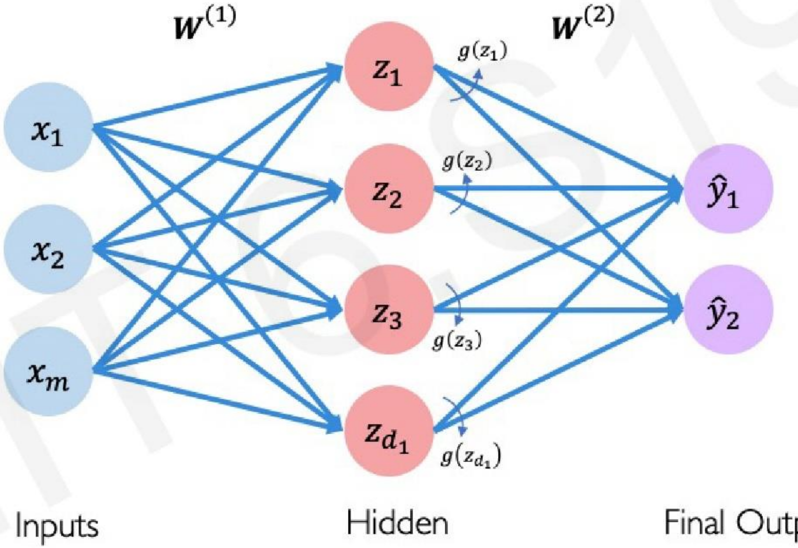



```python
import tensorflow as tf

layer = tf.keras.layers.Dense(
    units=2)
```

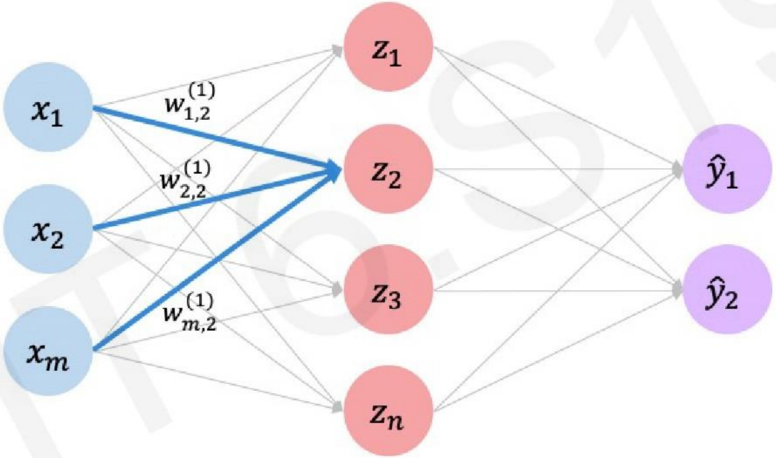
$$z_i = w_{0,i} + \sum_{j=1}^{m} x_j\, w_{j,i}$$

Single Layer Neural Network



$$z_i = w_{0,i}^{(1)} + \sum_{j=1}^{m} x_j\, w_{j,i}^{(1)} \qquad \hat{y}_i = g\left(w_{0,i}^{(2)} + \sum_{j=1}^{d_1} g(z_j)\, w_{j,i}^{(2)}\right)$$

Single Layer Neural Network



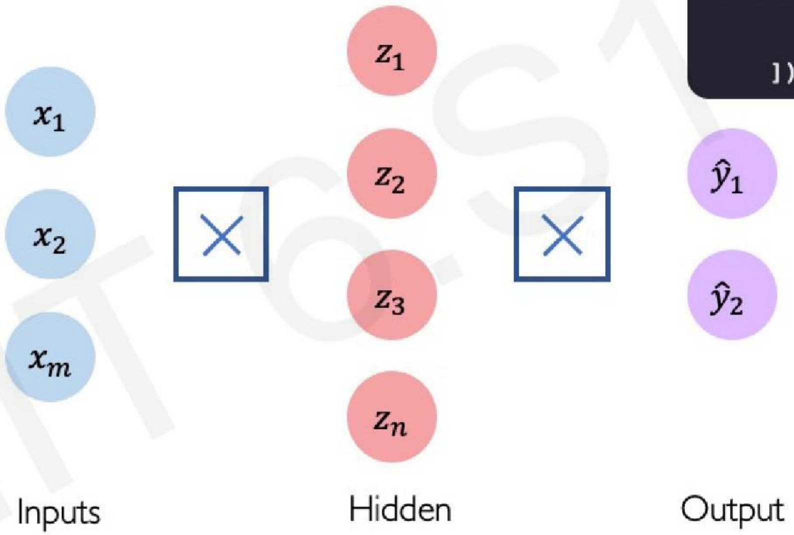
$$z_2 = w_{0,2}^{(1)} + \sum_{j=1}^{m} x_j\, w_{j,2}^{(1)}$$

$$= w_{0,2}^{(1)} + x_1\, w_{1,2}^{(1)} + x_2\, w_{2,2}^{(1)} + x_m\, w_{m,2}^{(1)}$$

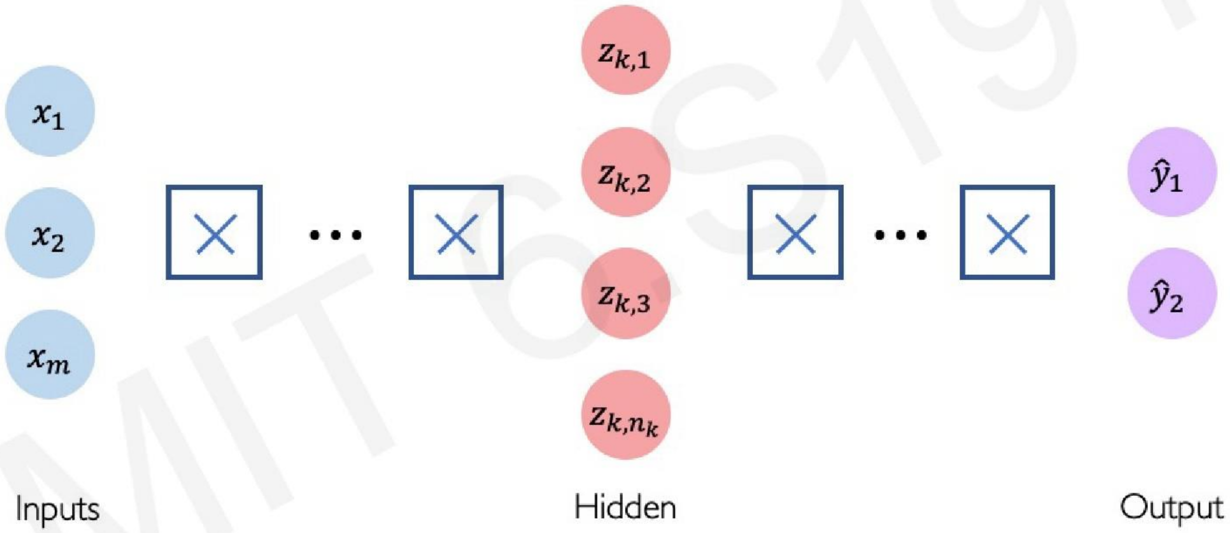
Multi Output Perceptron

```python
import tensorflow as tf

model = tf.keras.Sequential([
        tf.keras.layers.Dense(n),
        tf.keras.layers.Dense(2)
])
```

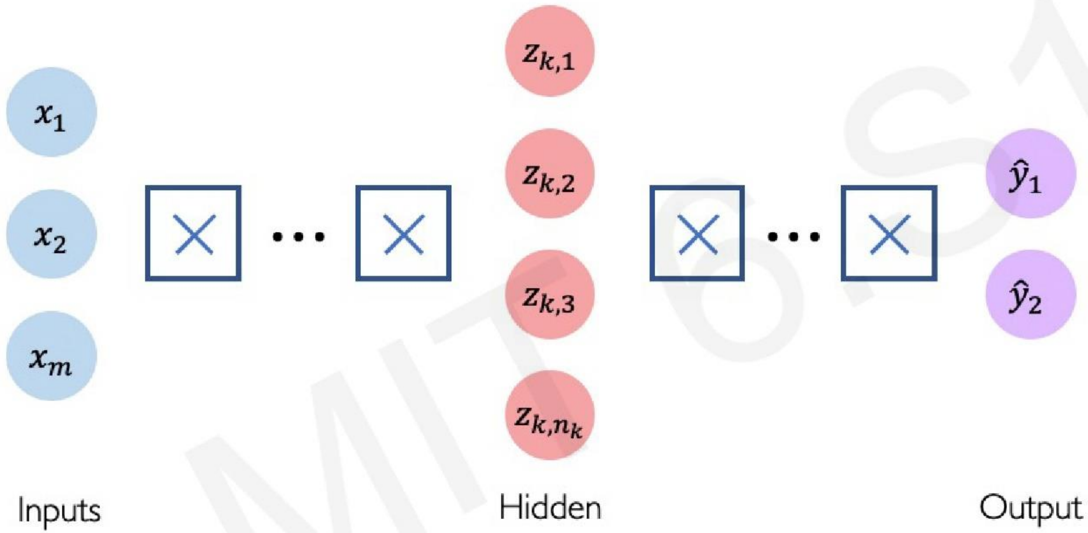


Deep Neural Network



$$z_{k,i} = w_{0,i}^{(k)} + \sum_{j=1}^{n_{k-1}} g(z_{k-1,j})\, w_{j,i}^{(k)}$$

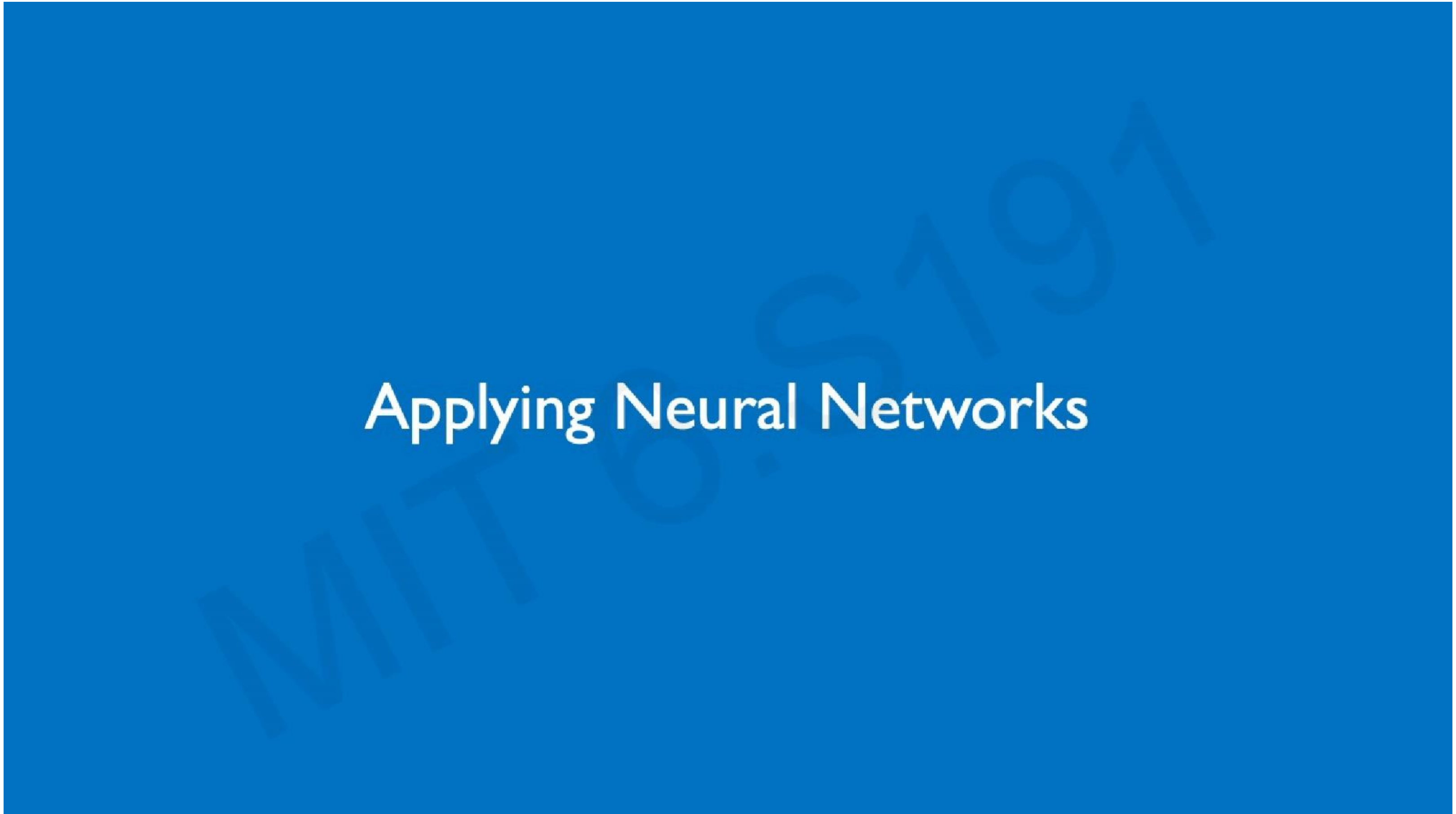
Deep Neural Network




```python
import tensorflow as tf

model = tf.keras.Sequential([
  tf.keras.layers.Dense(n₁),
  tf.keras.layers.Dense(n₂),
  ⋮
  tf.keras.layers.Dense(2)
])
```

$$z_{k,i} = w_{0,i}^{(k)} + \sum_{j=1}^{n_{k-1}} g(z_{k-1,j})\, w_{j,i}^{(k)}$$


Applying Neural Networks
MIT 6.S191

Example Problem

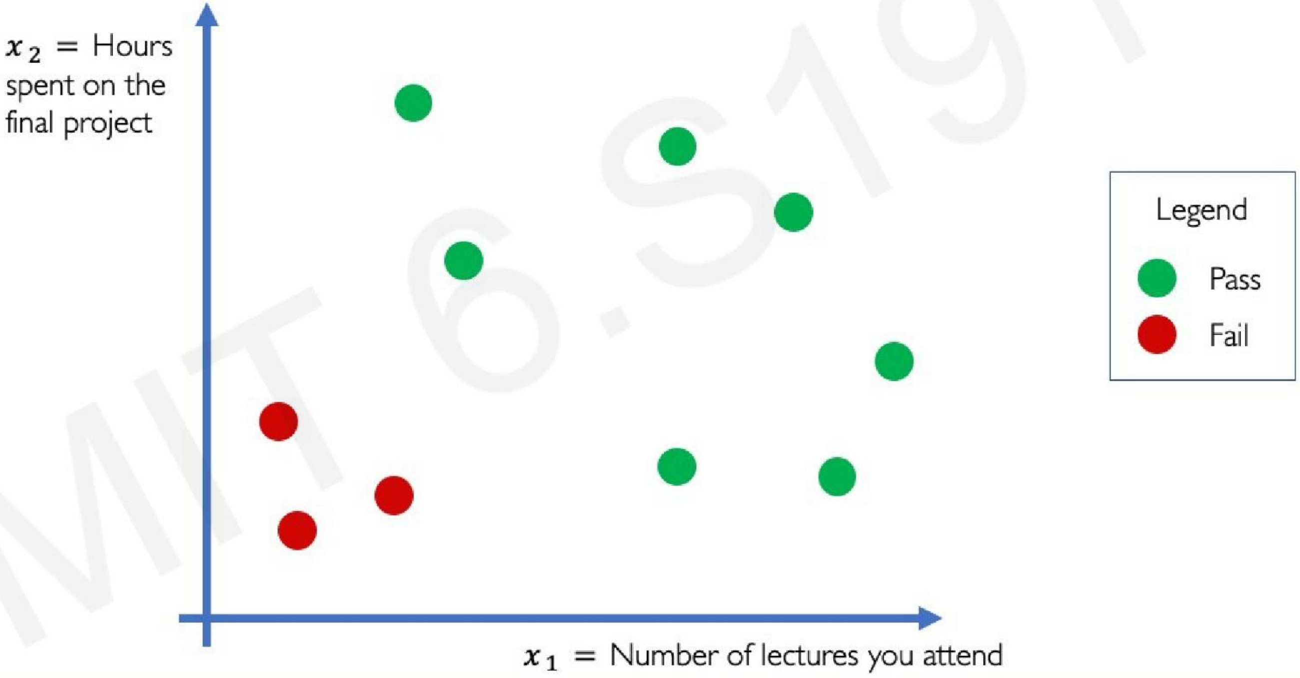
Will I pass this class?

Let's start with a simple two feature model

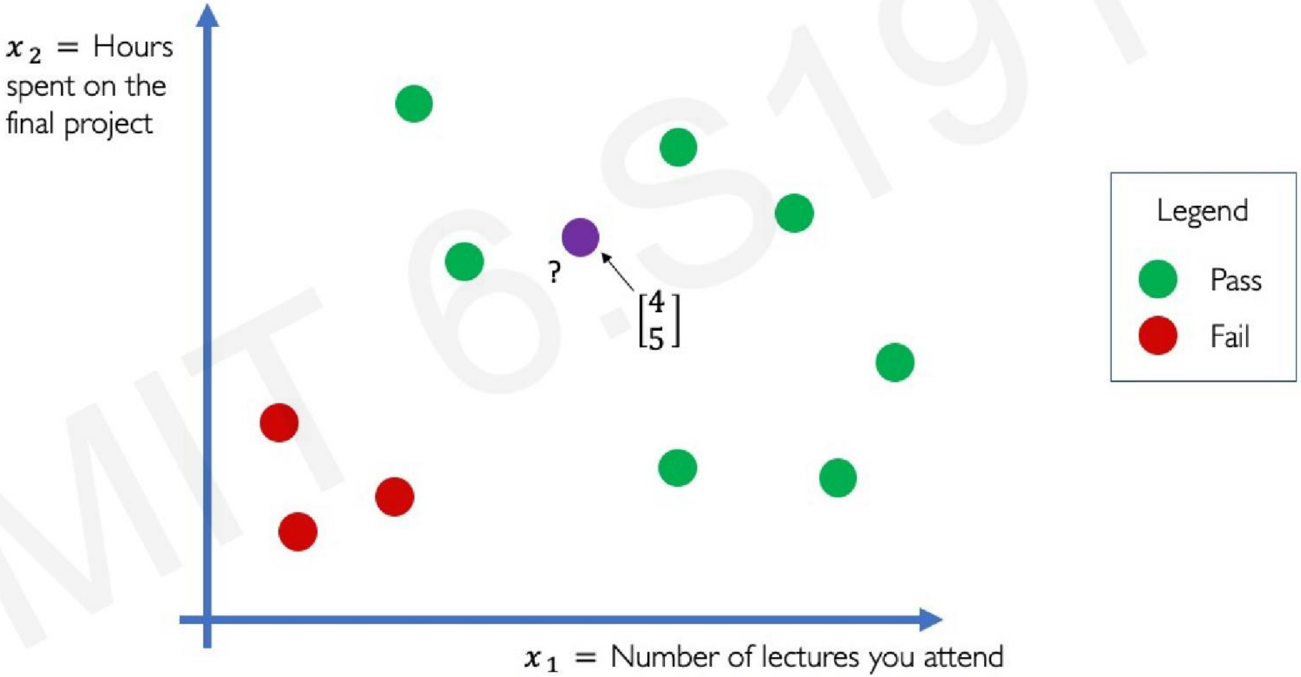
x_1 = Number of lectures you attend

x_2 = Hours spent on the final project

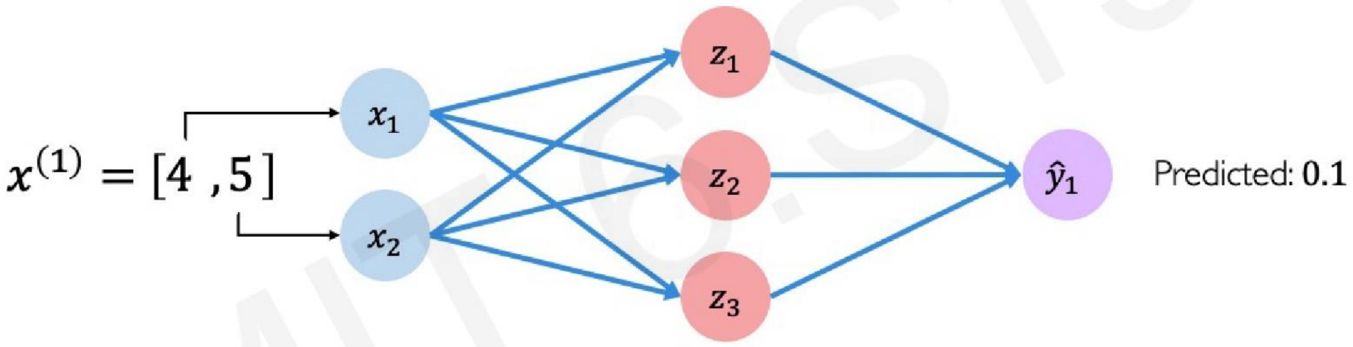
Example Problem: Will I pass this class?



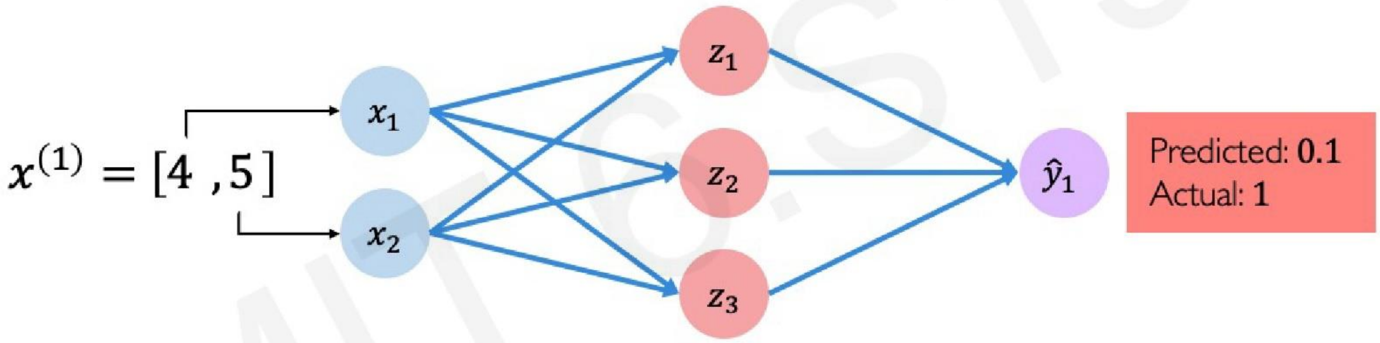
Example Problem: Will I pass this class?



Example Problem: Will I pass this class?

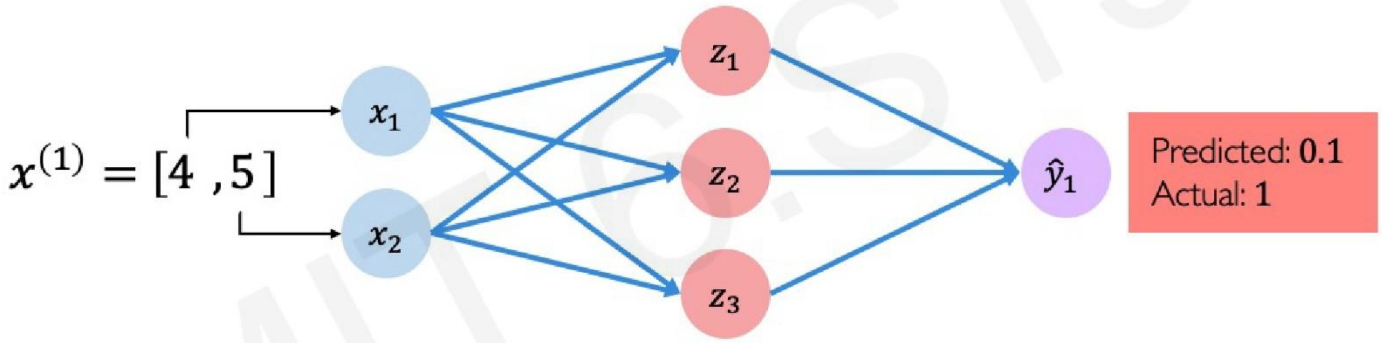


Example Problem: Will I pass this class?



Quantifying Loss

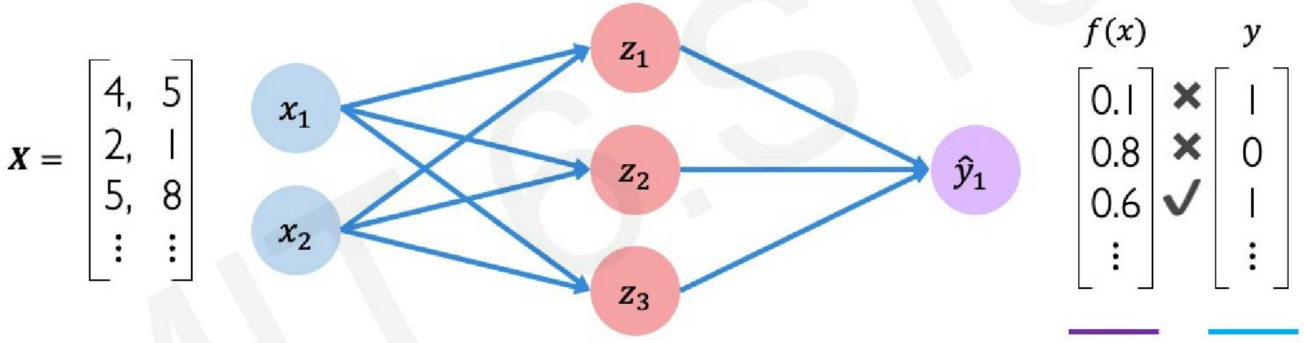
The ***loss*** *of our network measures the cost incurred from incorrect predictions*



$$\mathcal{L}\left(\underbrace{f\left(x^{(i)}; \boldsymbol{W}\right)}_{\text{Predicted}}, \underbrace{y^{(i)}}_{\text{Actual}}\right)$$

Empirical Loss

*The **empirical loss** measures the total loss over our entire dataset*



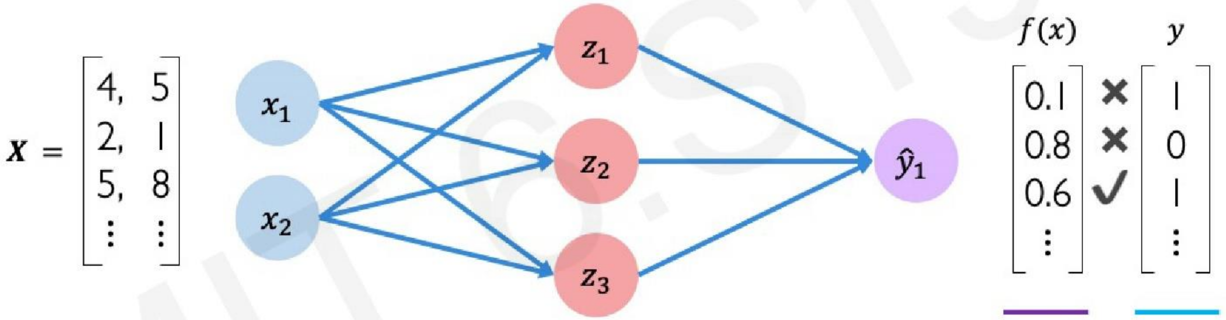
$$J(W) = \frac{1}{n}\sum_{i=1}^{n} \mathcal{L}\big(\underbrace{f\big(x^{(i)}; W\big)}_{\text{Predicted}}, \underbrace{y^{(i)}}_{\text{Actual}}\big)$$

Also known as:

- Objective function
- Cost function
- Empirical Risk

Binary Cross Entropy Loss

Cross entropy loss *can be used with models that output a probability between 0 and 1*

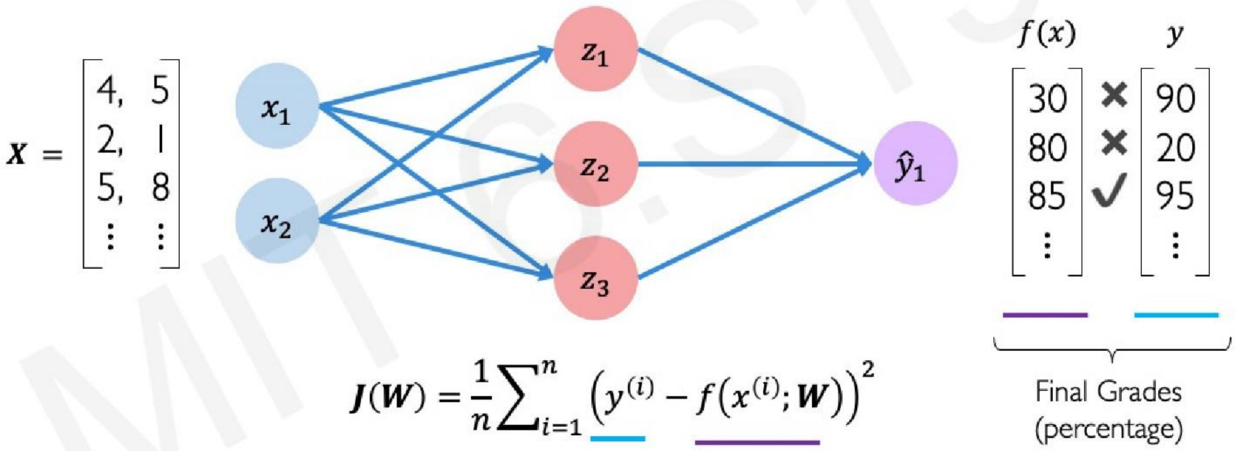


$$J(\boldsymbol{W}) = -\frac{1}{n}\sum_{i=1}^{n} \underbrace{y^{(i)}}_{\text{Actual}} \log\Big(\underbrace{f\big(x^{(i)};\boldsymbol{W}\big)}_{\text{Predicted}}\Big) + (1-\underbrace{y^{(i)}}_{\text{Actual}}) \log\Big(1-\underbrace{f\big(x^{(i)};\boldsymbol{W}\big)}_{\text{Predicted}}\Big)$$

```
loss = tf.reduce_mean( tf.nn.softmax_cross_entropy_with_logits(y, predicted) )
```

Mean Squared Error Loss

***Mean squared error loss** can be used with regression models that output continuous real numbers*



$$J(W) = \frac{1}{n}\sum_{i=1}^{n}\left(\underbrace{y^{(i)}}_{\text{Actual}} - \underbrace{f\left(x^{(i)}; W\right)}_{\text{Predicted}}\right)^2$$

```
loss = tf.reduce_mean( tf.square(tf.subtract(y, predicted)) )
loss = tf.keras.losses.MSE( y, predicted )
```

Training Neural Networks

Loss Optimization

We want to find the network weights that ***achieve the lowest loss***

$$\boldsymbol{W}^* = \underset{\boldsymbol{W}}{\operatorname{argmin}} \frac{1}{n} \sum_{i=1}^{n} \mathcal{L}\left(f\left(x^{(i)}; \boldsymbol{W}\right), y^{(i)}\right)$$

$$\boldsymbol{W}^* = \underset{\boldsymbol{W}}{\operatorname{argmin}}\, J(\boldsymbol{W})$$

Loss Optimization

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$$\boldsymbol{W}^* = \underset{\boldsymbol{W}}{\operatorname{argmin}}\, J(\boldsymbol{W})$$

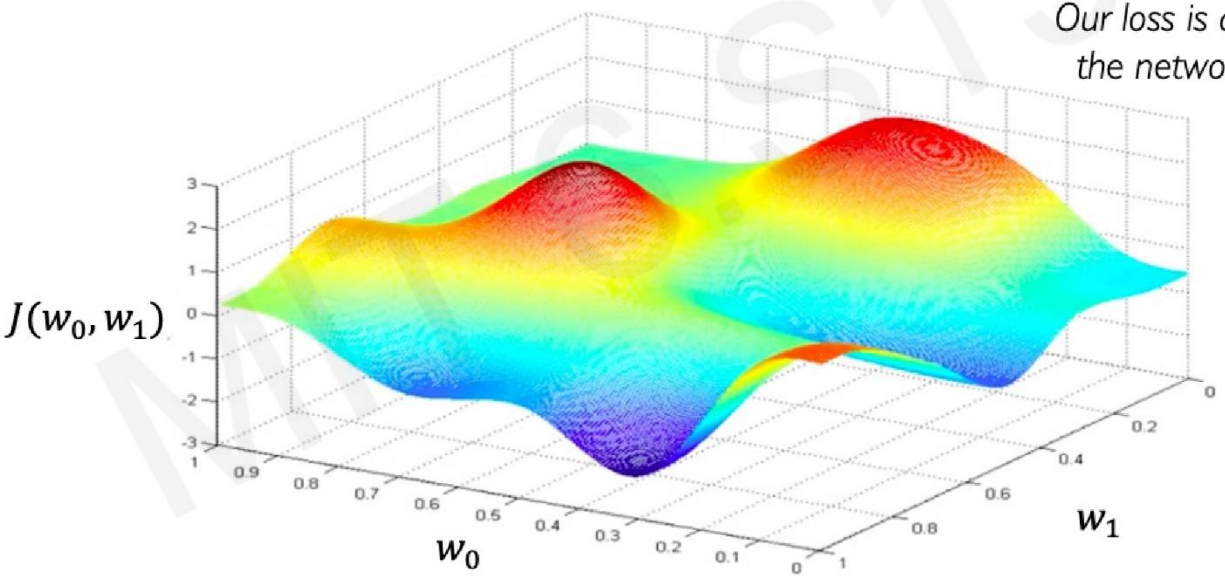
Remember:

$$\boldsymbol{W} = \left\{\boldsymbol{W}^{(0)}, \boldsymbol{W}^{(1)}, \cdots\right\}$$

Loss Optimization

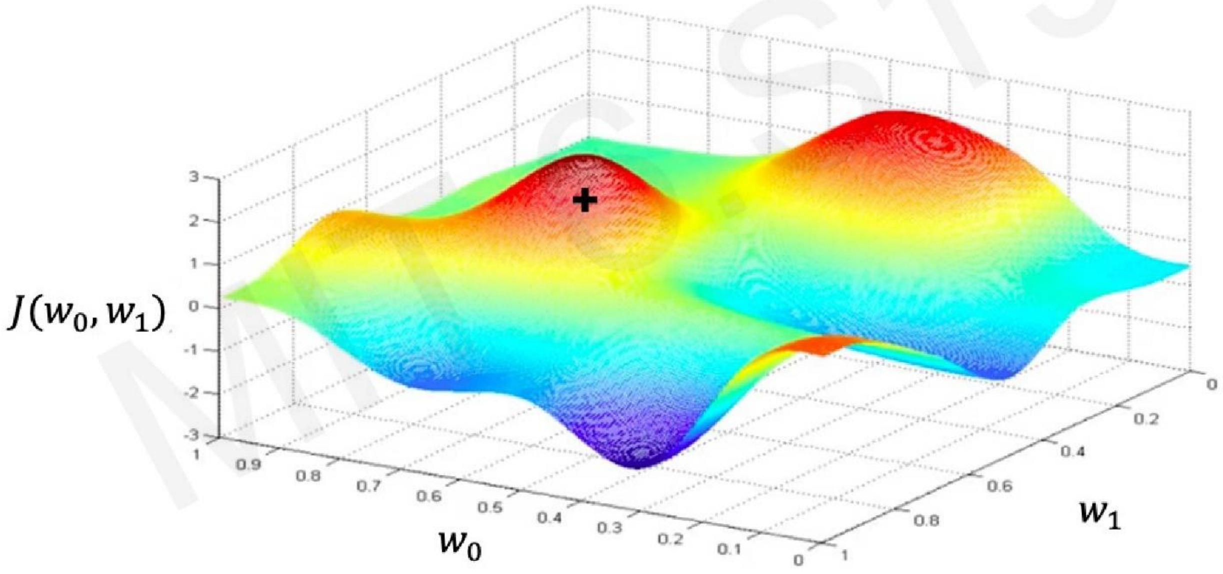
$$W^* = \underset{W}{\operatorname{argmin}}\, J(W)$$

Remember:
Our loss is a function of the network weights!

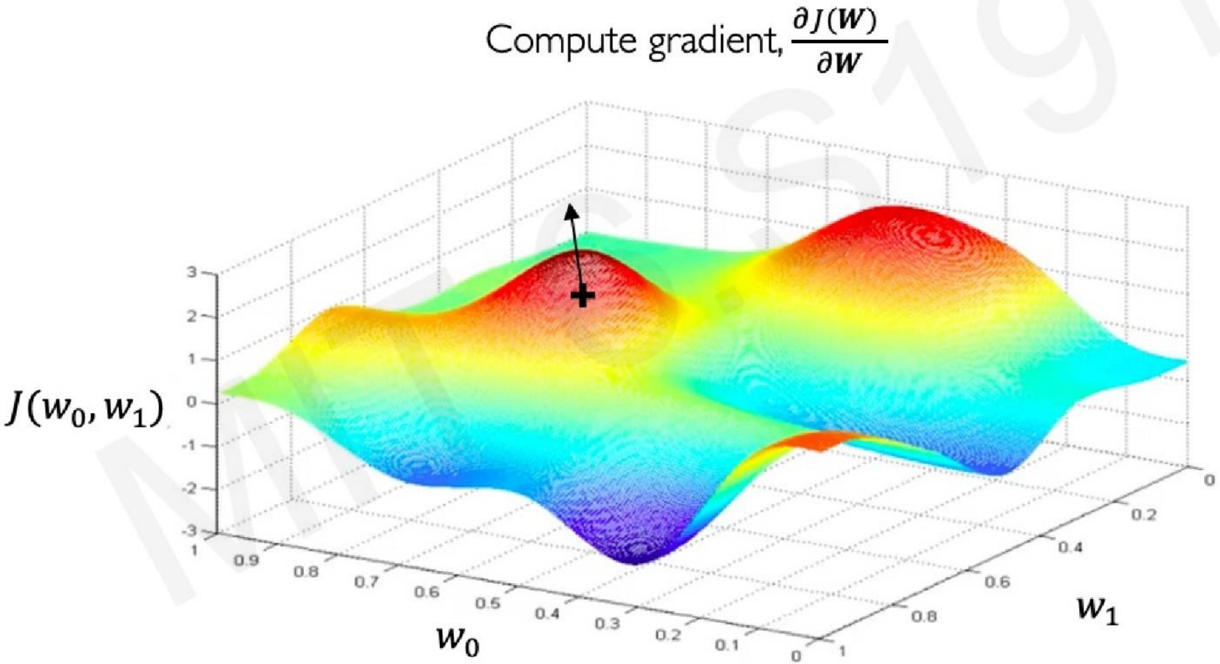


Loss Optimization

Randomly pick an initial (w_0, w_1)

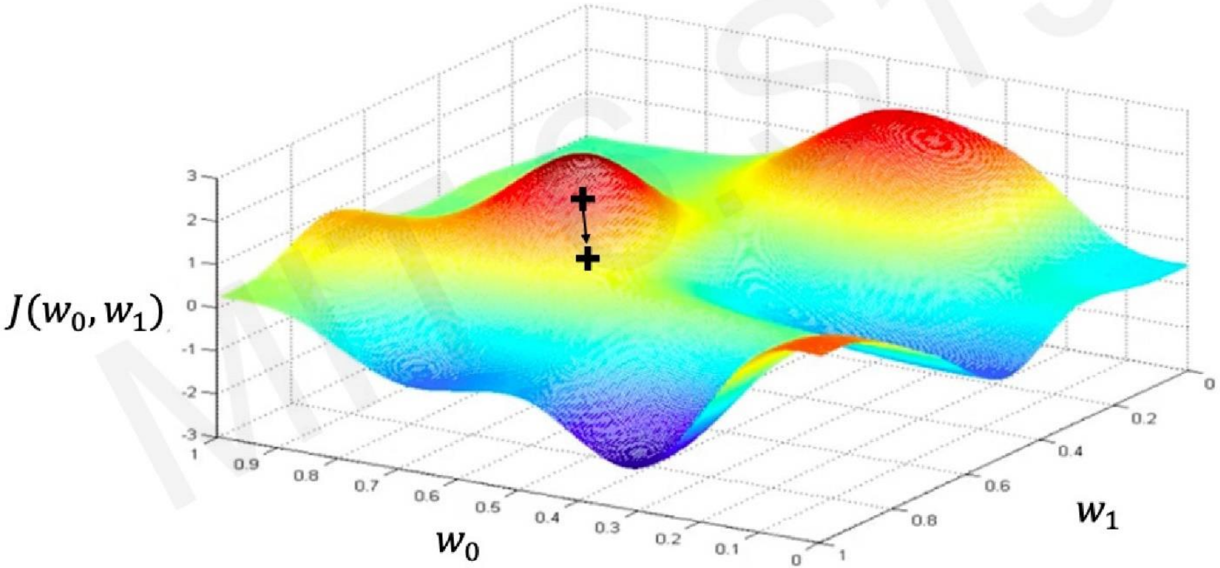


Loss Optimization



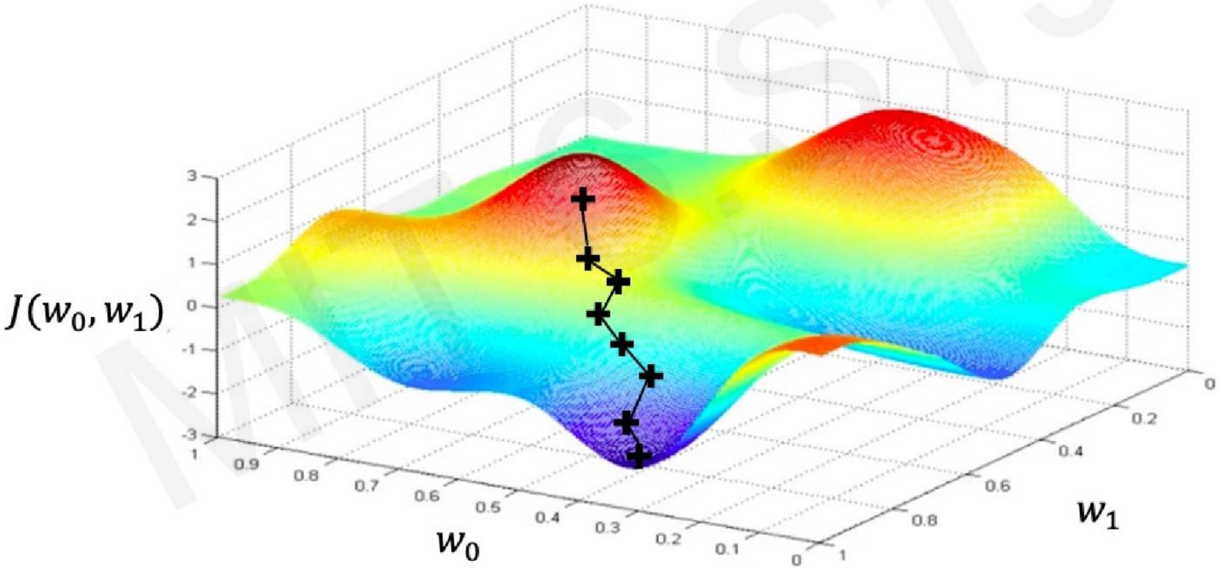
Loss Optimization

Take small step in opposite direction of gradient



Gradient Descent

Repeat until convergence



Gradient Descent

Algorithm

1. Initialize weights randomly $\sim\mathcal{N}(0, \sigma^2)$
2. Loop until convergence:
3. Compute gradient, $\frac{\partial J(\boldsymbol{W})}{\partial \boldsymbol{W}}$
4. Update weights, $\boldsymbol{W} \leftarrow \boldsymbol{W} - \eta \frac{\partial J(\boldsymbol{W})}{\partial \boldsymbol{W}}$
5. Return weights

Gradient Descent

Algorithm

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5. Return weights

```python
import tensorflow as tf

weights = tf.Variable([tf.random.normal()])

while True:    # loop forever
    with tf.GradientTape() as g:
        loss = compute_loss(weights)
        gradient = g.gradient(loss, weights)

    weights = weights - lr * gradient
```

Gradient Descent

Algorithm

1. Initialize weights randomly $\sim\mathcal{N}(0, \sigma^2)$
2. Loop until convergence:
3. Compute gradient, $\frac{\partial J(W)}{\partial W}$
4. Update weights, $W \leftarrow W - \eta \frac{\partial J(W)}{\partial W}$
5. Return weights

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```

Computing Gradients: Backpropagation

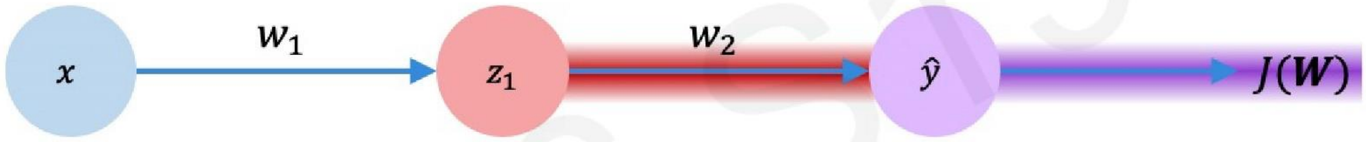
How does a small change in one weight (ex. $\boldsymbol{w_2}$*) affect the final loss* $J(\boldsymbol{W})$*?*

Computing Gradients: Backpropagation

$$\frac{\partial J(\boldsymbol{W})}{\partial \boxed{w_2}} =$$

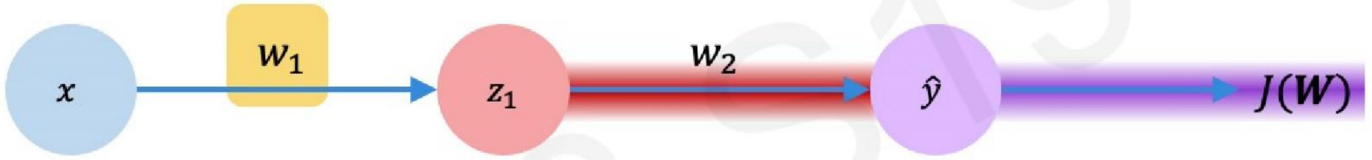
Let's use the chain rule!

Computing Gradients: Backpropagation



$$\frac{\partial J(\boldsymbol{W})}{\partial w_2} = \frac{\partial J(\boldsymbol{W})}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial w_2}$$

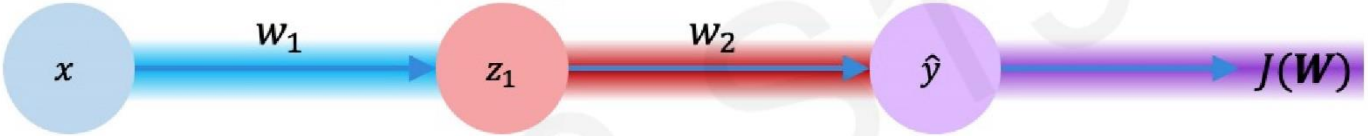
Computing Gradients: Backpropagation



$$\frac{\partial J(\boldsymbol{W})}{\partial w_1} = \frac{\partial J(\boldsymbol{W})}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial w_1}$$

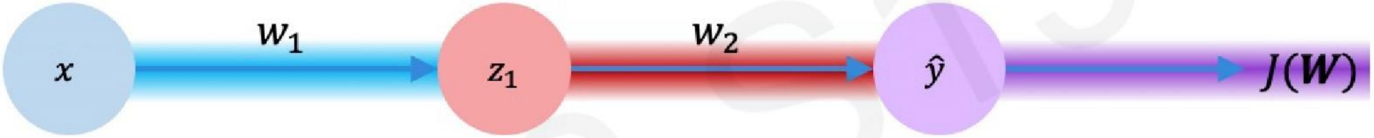
Apply chain rule!　　Apply chain rule!

Computing Gradients: Backpropagation



$$\frac{\partial J(\boldsymbol{W})}{\partial w_1} = \frac{\partial J(\boldsymbol{W})}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial z_1} * \frac{\partial z_1}{\partial w_1}$$

Computing Gradients: Backpropagation

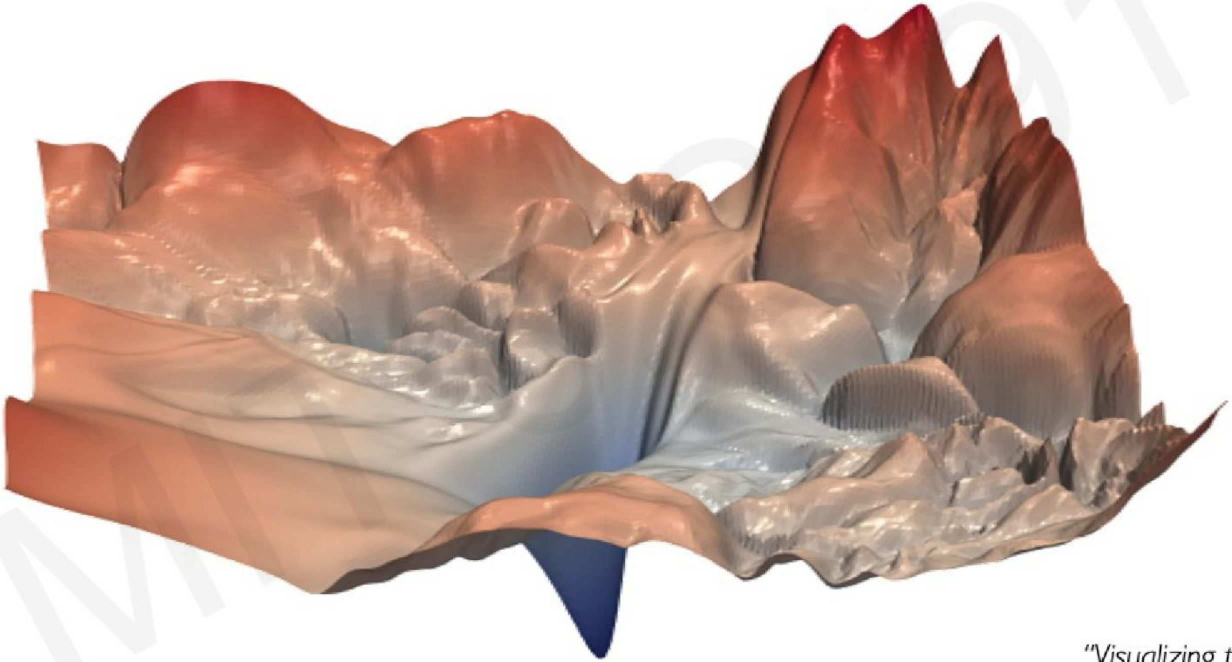


$$\frac{\partial J(\boldsymbol{W})}{\partial w_1} = \frac{\partial J(\boldsymbol{W})}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial z_1} * \frac{\partial z_1}{\partial w_1}$$

Repeat this for ***every weight in the network*** *using gradients from later layers*

Neural Networks in Practice: Optimization

Training Neural Networks is Difficult



"Visualizing the loss landscape of neural nets". Dec 2017.

Loss Functions Can Be Difficult to Optimize

Remember:

Optimization through gradient descent

$$\boldsymbol{W} \leftarrow \boldsymbol{W} - \eta \frac{\partial J(\boldsymbol{W})}{\partial \boldsymbol{W}}$$

Loss Functions Can Be Difficult to Optimize

Remember:
Optimization through gradient descent

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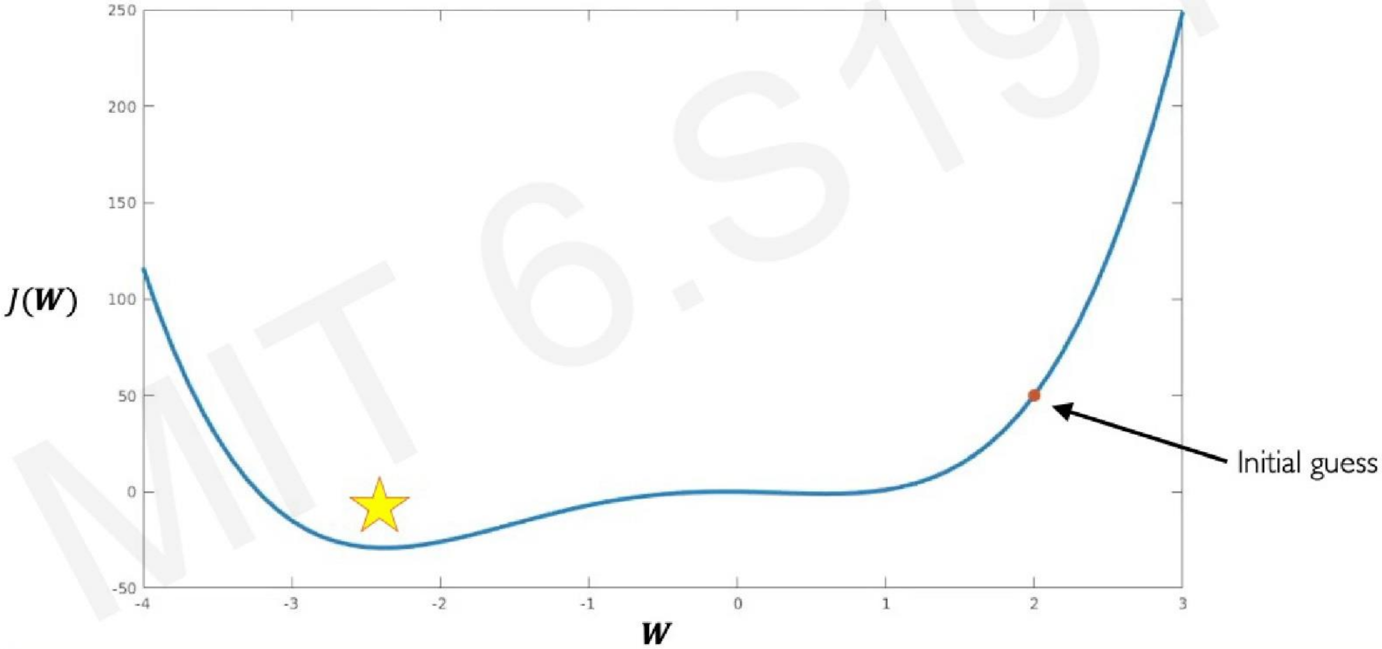
How can we set the learning rate?

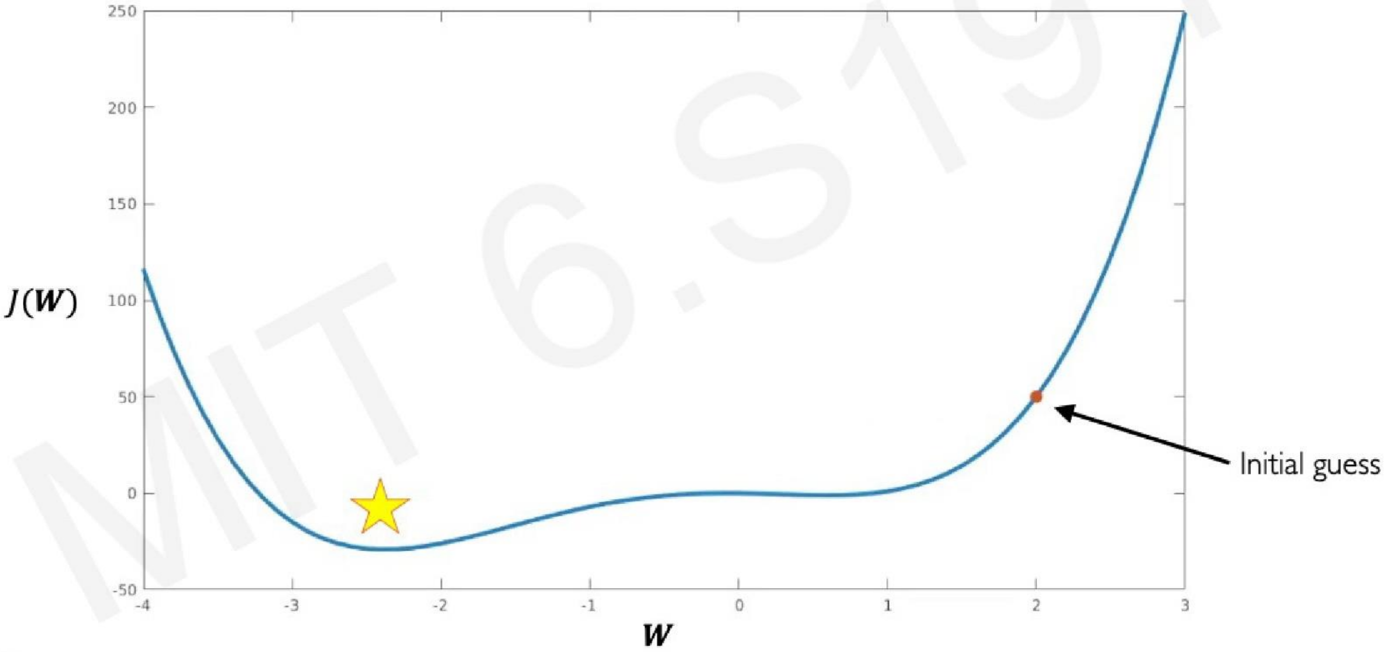
Setting the Learning Rate

Small learning rate *converges slowly and gets stuck in false local minima*



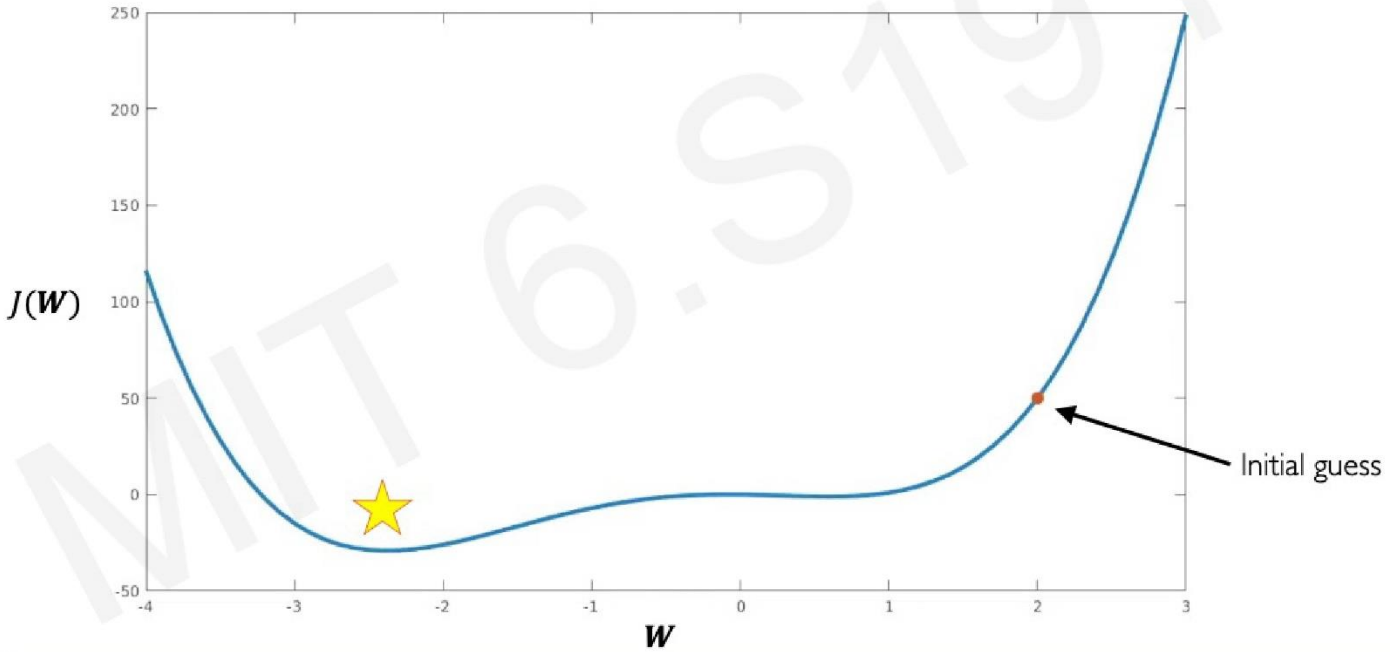
Setting the Learning Rate

***Large learning rates** overshoot, become unstable and diverge*



Setting the Learning Rate

Stable learning rates *converge smoothly and avoid local minima*



How to deal with this?

Idea 1:

Try lots of different learning rates and see what works "just right"

How to deal with this?

Idea 1:

Try lots of different learning rates and see what works "just right"

Idea 2:

Do something smarter!
Design an adaptive learning rate that "adapts" to the landscape

Adaptive Learning Rates

- Learning rates are no longer fixed
- Can be made larger or smaller depending on:
  - how large gradient is
  - how fast learning is happening
  - size of particular weights
  - etc...

Gradient Descent Algorithms

Algorithm	TF Implementation	Reference
• SGD	`tf.keras.optimizers.SGD`	Kiefer & Wolfowitz. "Stochastic Estimation of the Maximum of a Regression Function." 1952.
• Adam	`tf.keras.optimizers.Adam`	Kingma et al. "Adam: A Method for Stochastic Optimization." 2014.
• Adadelta	`tf.keras.optimizers.Adadelta`	Zeiler et al. "ADADELTA: An Adaptive Learning Rate Method." 2012.
• Adagrad	`tf.keras.optimizers.Adagrad`	Duchi et al. "Adaptive Subgradient Methods for Online Learning and Stochastic Optimization." 2011.
• RMSProp	`tf.keras.optimizers.RMSProp`	

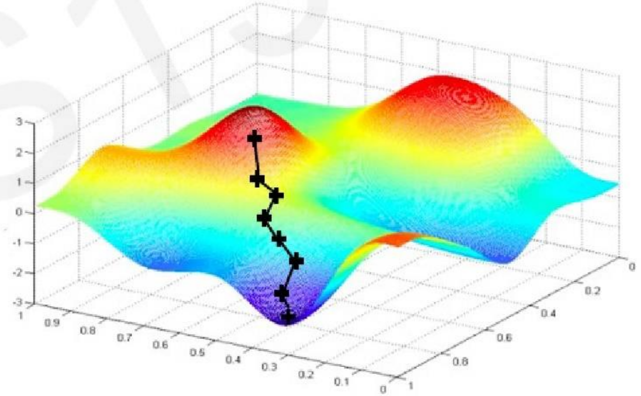
Additional details: http://ruder.io/optimizing-gradient-descent/

Neural Networks in Practice: Mini-batches

Gradient Descent

Algorithm

1. Initialize weights randomly $\sim\mathcal{N}(0, \sigma^2)$
2. Loop until convergence:
3. Compute gradient, $\frac{\partial J(\boldsymbol{W})}{\partial \boldsymbol{W}}$
4. Update weights, $\boldsymbol{W} \leftarrow \boldsymbol{W} - \eta \frac{\partial J(\boldsymbol{W})}{\partial \boldsymbol{W}}$
5. Return weights



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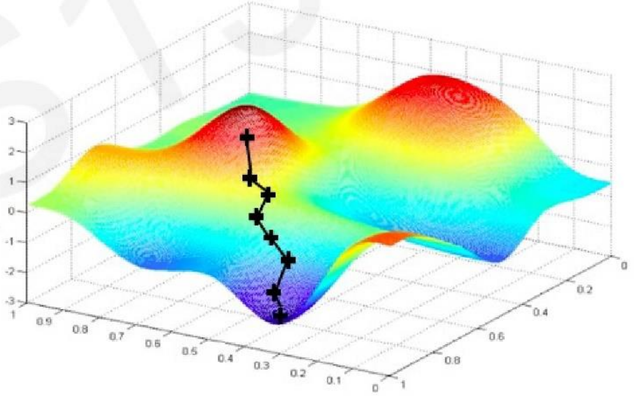
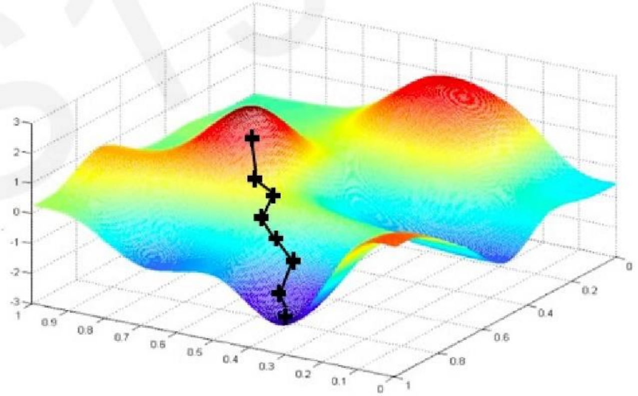


Can be very **computationally intensive** to compute!

Stochastic Gradient Descent

Algorithm

1. Initialize weights randomly $\sim\mathcal{N}(0, \sigma^2)$
2. Loop until convergence:
3. Pick single data point $\boldsymbol{i}$
4. Compute gradient, $\frac{\partial J_i(\boldsymbol{W})}{\partial \boldsymbol{W}}$
5. Update weights, $\boldsymbol{W} \leftarrow \boldsymbol{W} - \eta \frac{\partial J(\boldsymbol{W})}{\partial \boldsymbol{W}}$
6. Return weights



Stochastic Gradient Descent

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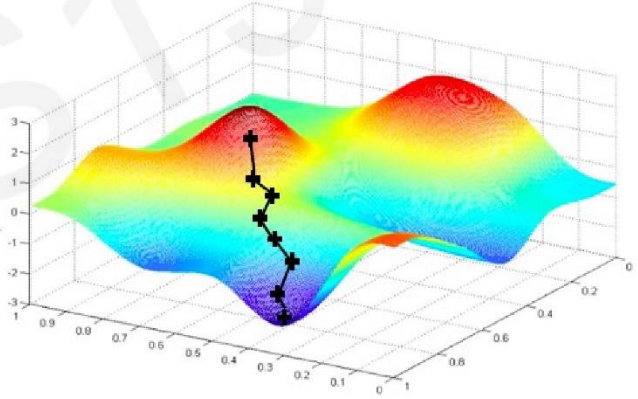

Easy to compute but
very noisy (stochastic)!

Stochastic Gradient Descent

Algorithm

1. Initialize weights randomly $\sim\mathcal{N}(0, \sigma^2)$
2. Loop until convergence:
3. Pick batch of B data points
4. Compute gradient, $\frac{\partial J(\boldsymbol{W})}{\partial \boldsymbol{W}} = \frac{1}{B}\sum_{k=1}^{B} \frac{\partial J_k(\boldsymbol{W})}{\partial \boldsymbol{W}}$
5. Update weights, $\boldsymbol{W} \leftarrow \boldsymbol{W} - \eta \frac{\partial J(\boldsymbol{W})}{\partial \boldsymbol{W}}$
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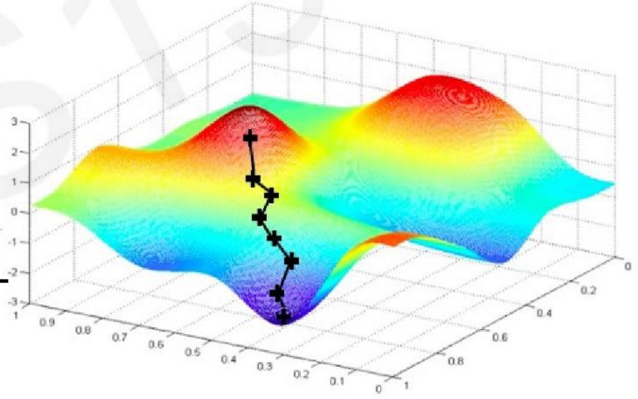

Stochastic Gradient Descent

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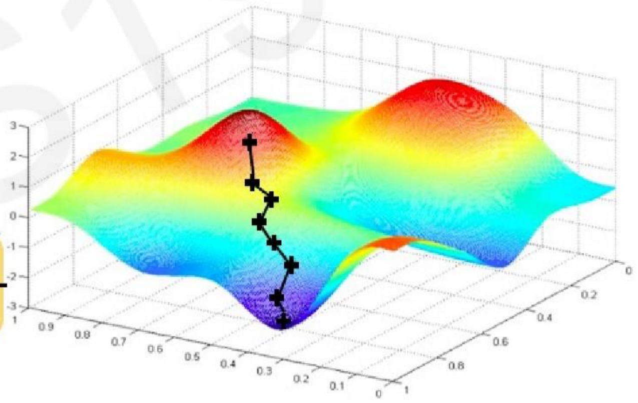


Fast to compute and a much better estimate of the true gradient!

Mini-batches while training

More accurate estimation of gradient

Smoother convergence

Allows for larger learning rates

Mini-batches while training

More accurate estimation of gradient

Smoother convergence

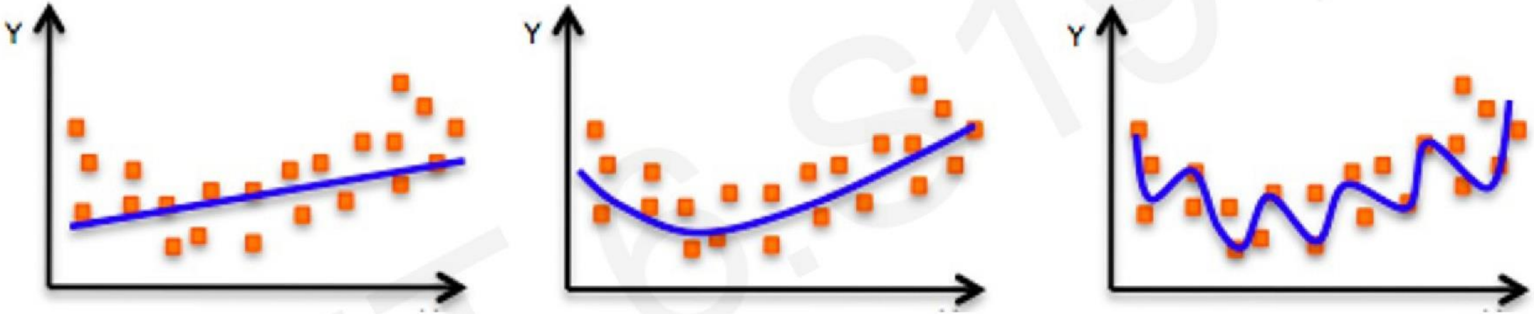
Allows for larger learning rates

Mini-batches lead to fast training!

Can parallelize computation + achieve significant speed increases on GPU's

Neural Networks in Practice: Overfitting

The Problem of Overfitting



Underfitting
Model does not have capacity to fully learn the data

← **Ideal fit** →

Overfitting
Too complex, extra parameters, does not generalize well

Regularization

What is it?

Technique that constrains our optimization problem to discourage complex models

Regularization

What is it?

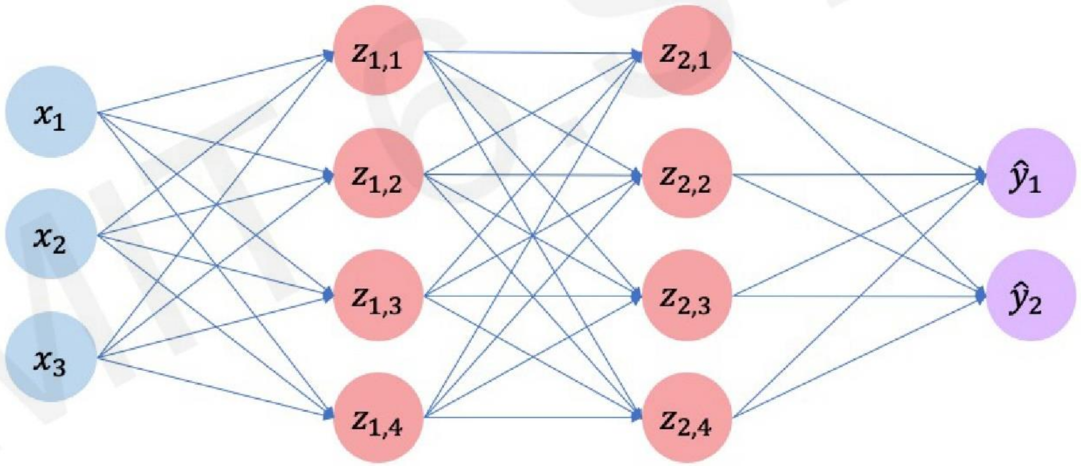
Technique that constrains our optimization problem to discourage complex models

Why do we need it?

Improve generalization of our model on unseen data

Regularization 1: Dropout

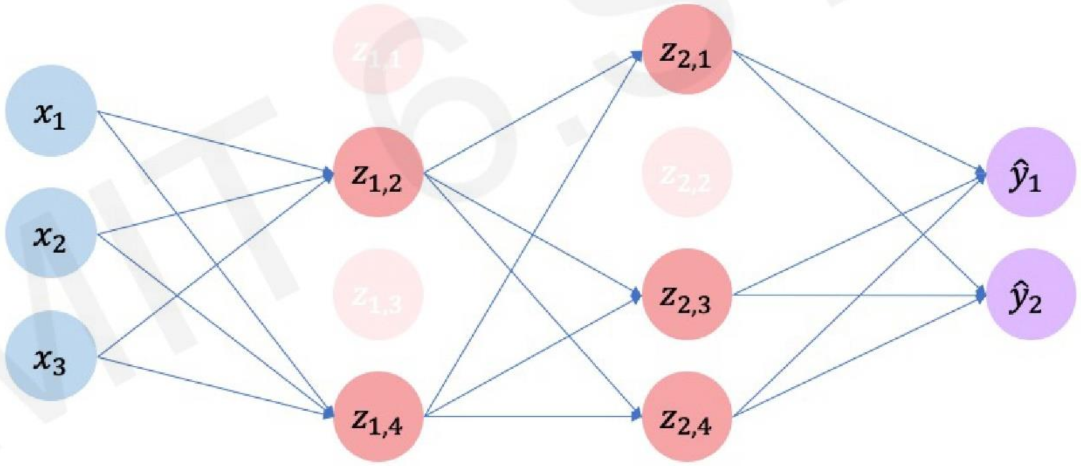
- During training, randomly set some activations to 0



Regularization 1: Dropout

- During training, randomly set some activations to 0
  - Typically 'drop' 50% of activations in layer
  - Forces network to not rely on any 1 node

```
tf.keras.layers.Dropout(p=0.5)
```



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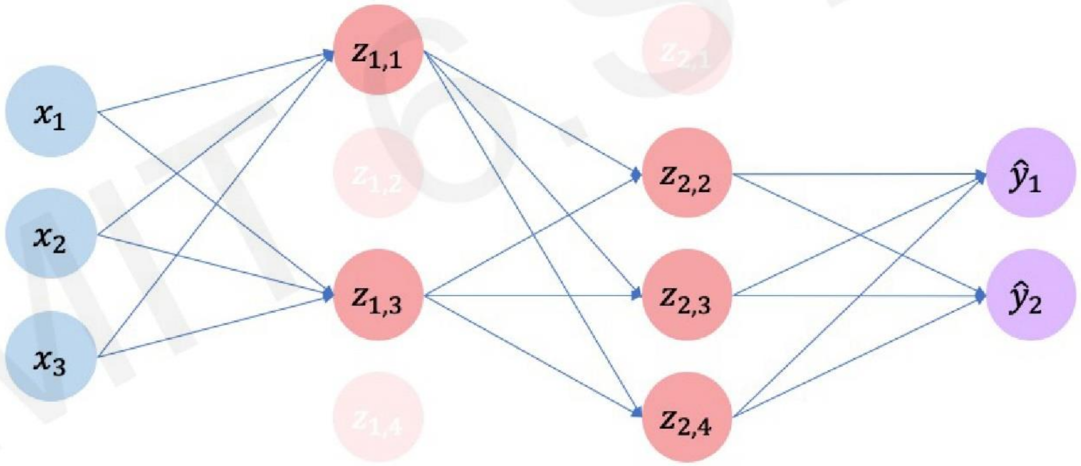

Regularization 2: Early Stopping

- Stop training before we have a chance to overfit

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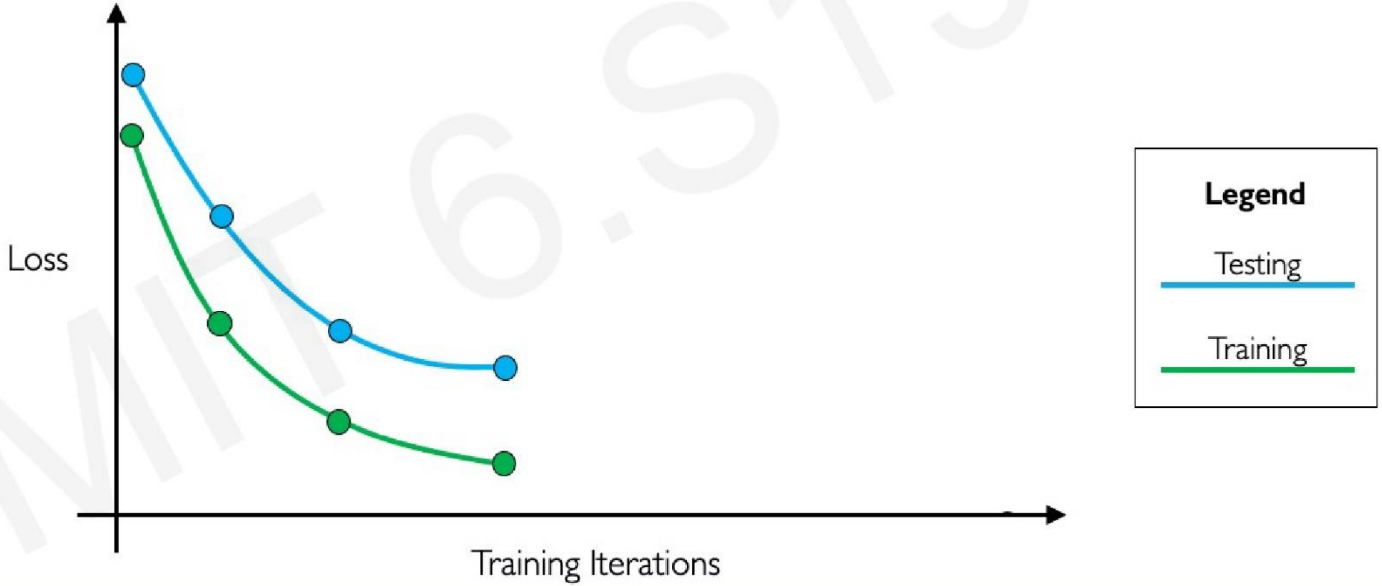

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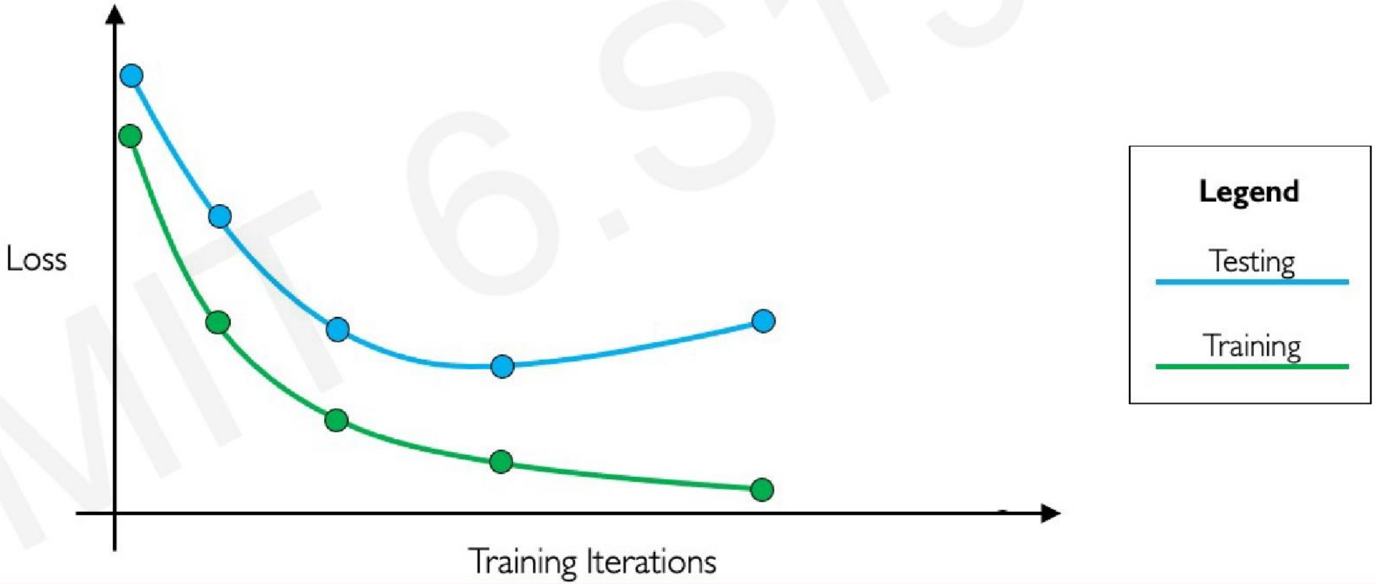

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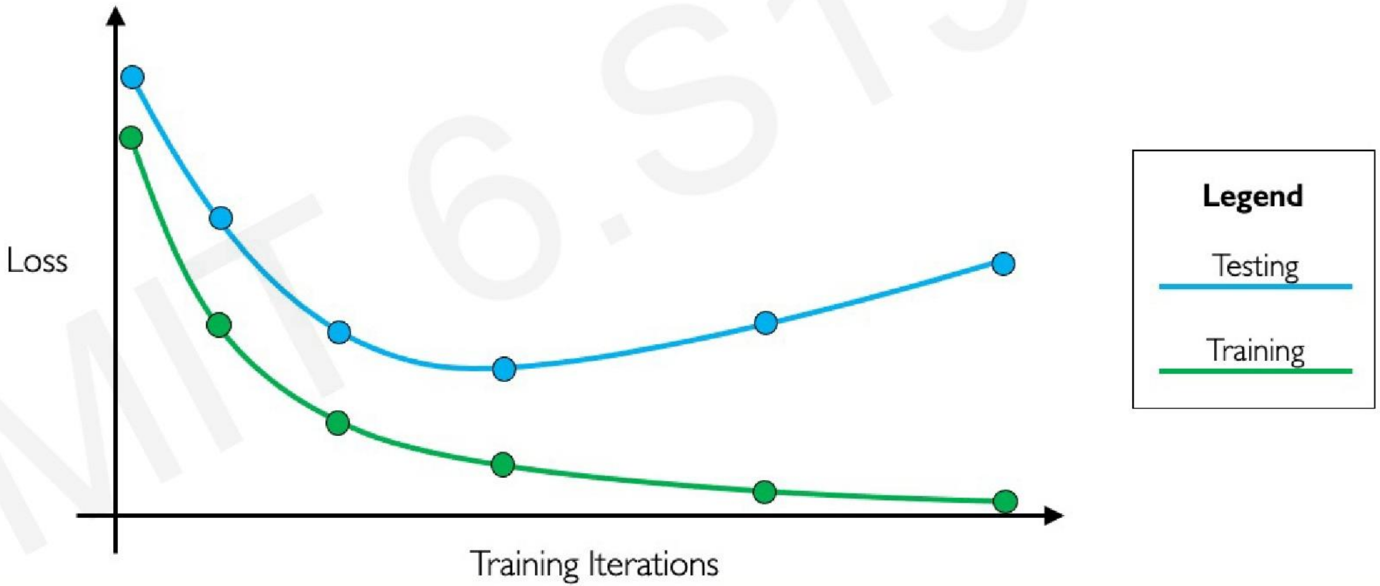

Regularization 2: Early Stopping

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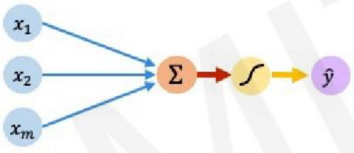
Core Foundation Review

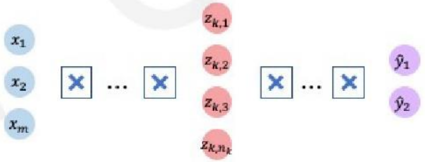
The Perceptron

- Structural building blocks
- Nonlinear activation functions



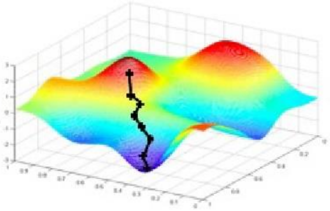
Neural Networks

- Stacking Perceptrons to form neural networks
- Optimization through backpropagation



Training in Practice

- Adaptive learning
- Batching
- Regularization



提纲

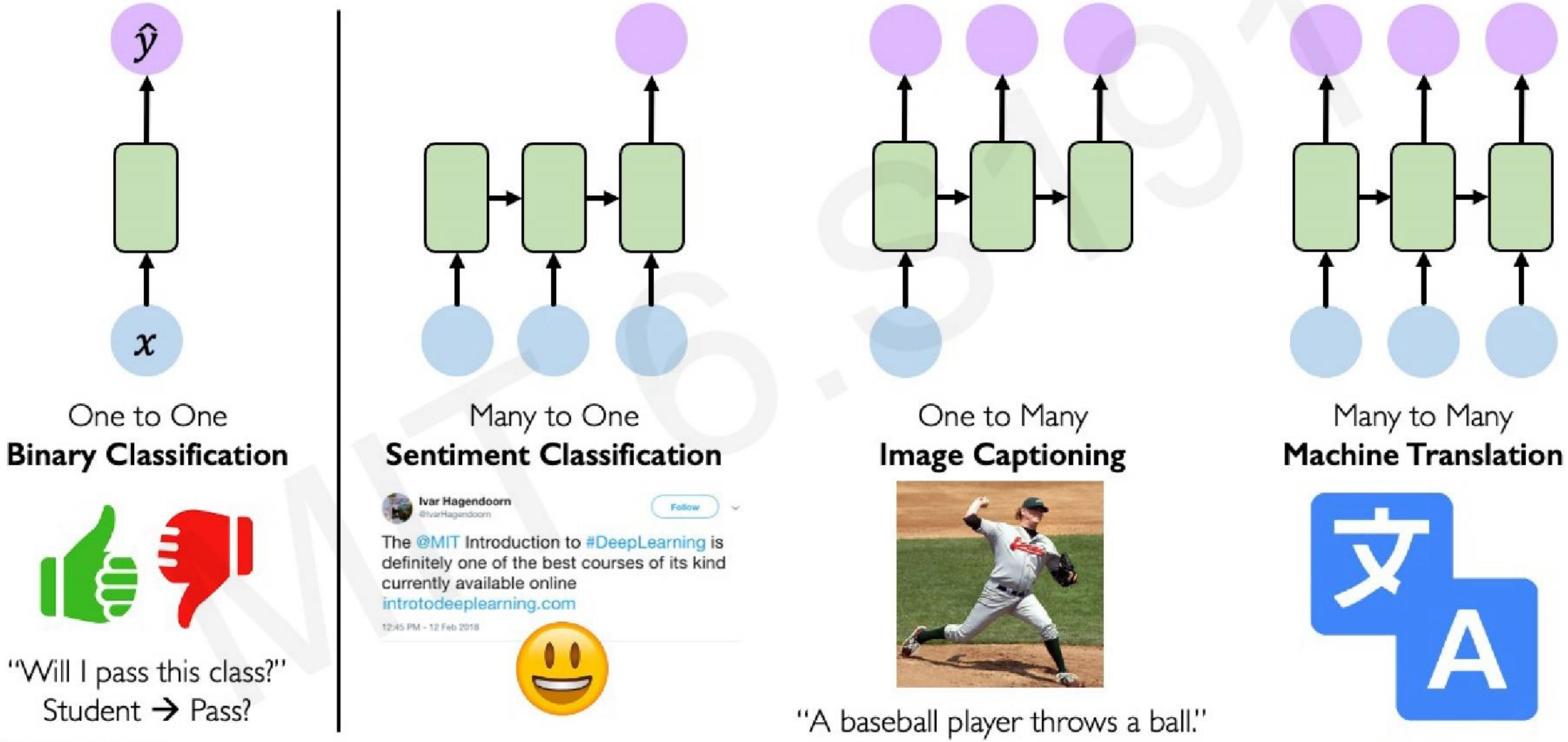
一、前馈网络

二、循环网络

三、卷积网络

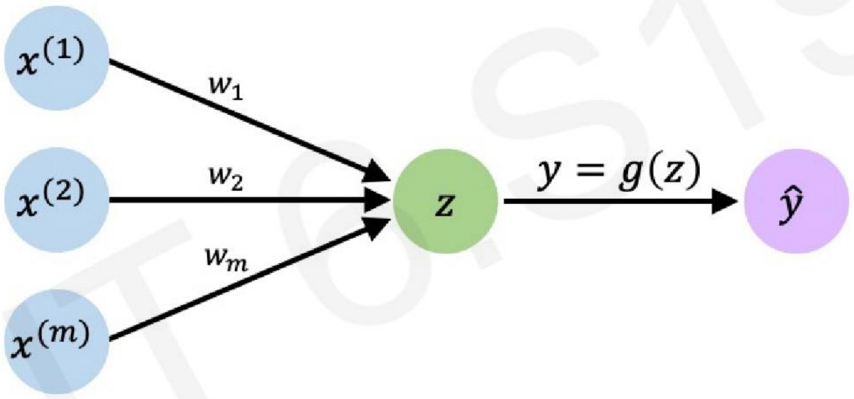
Sequences in the Wild

Sequence Modeling Applications

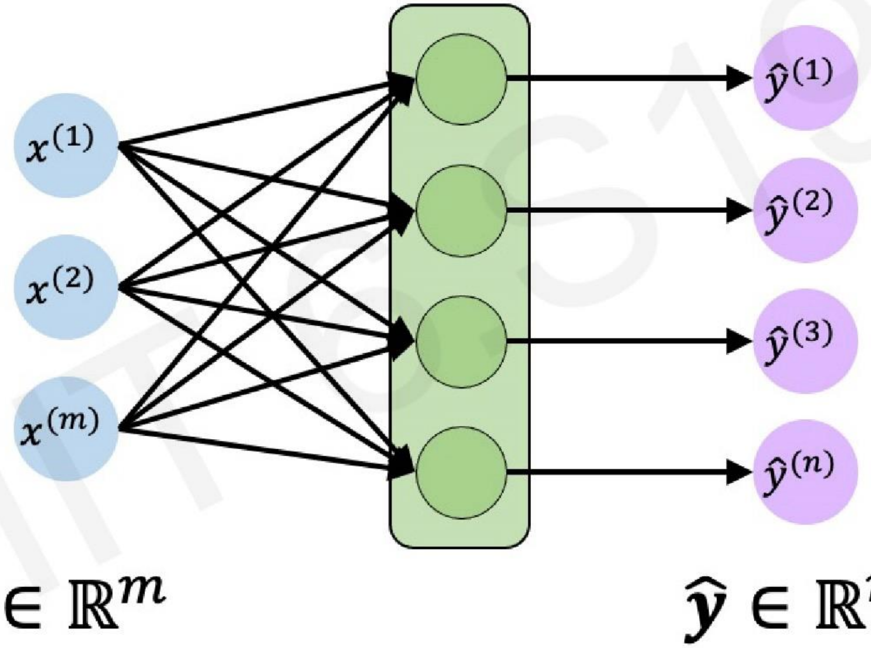


Neurons with Recurrence

The Perceptron Revisited



Feed-Forward Networks Revisited



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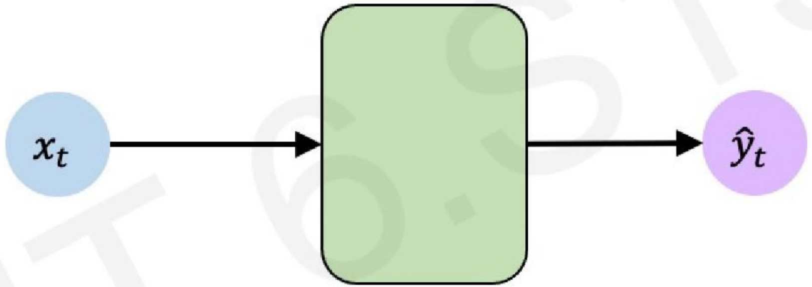
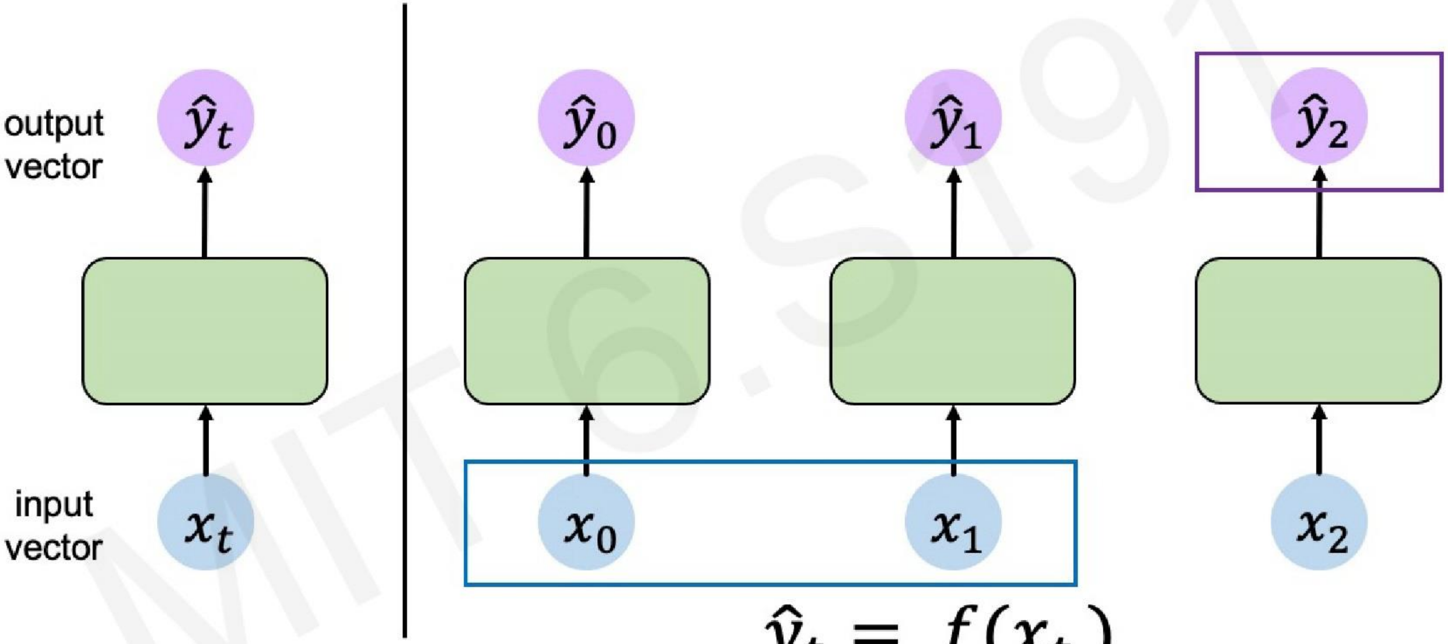


$\boldsymbol{x_t} \in \mathbb{R}^m$

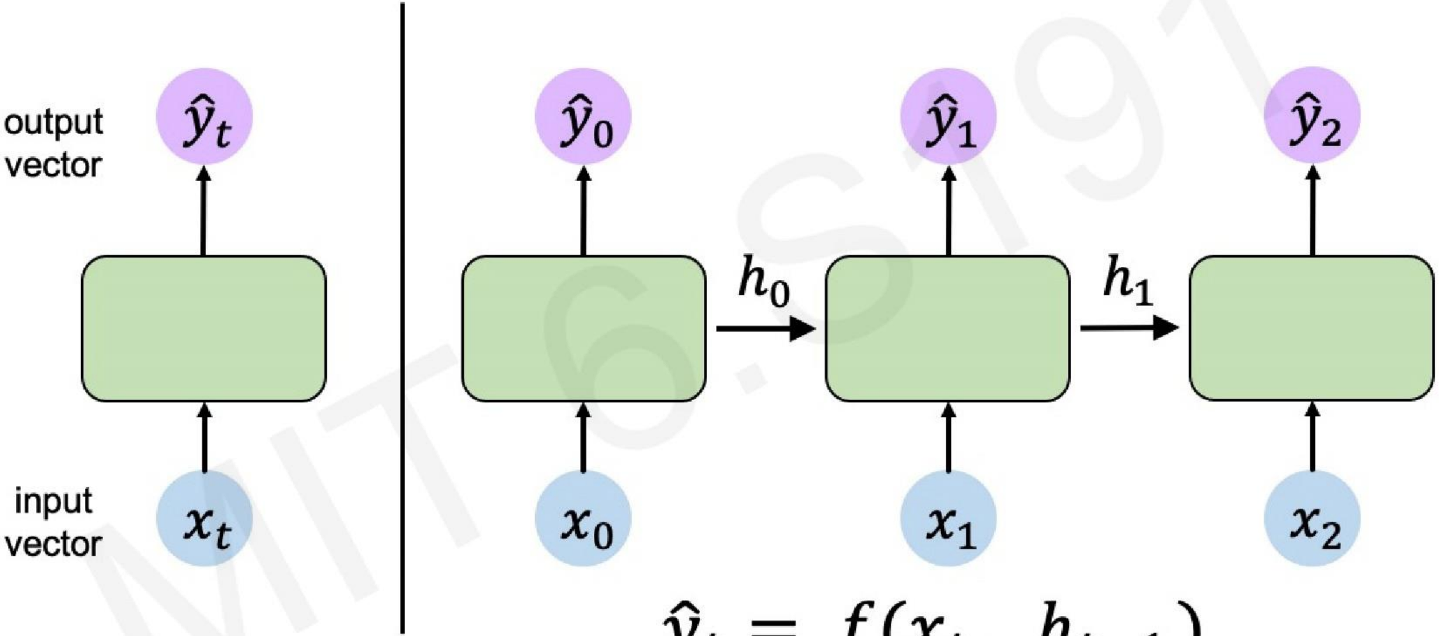
$\hat{\boldsymbol{y}}_t \in \mathbb{R}^n$

Handling Individual Time Steps



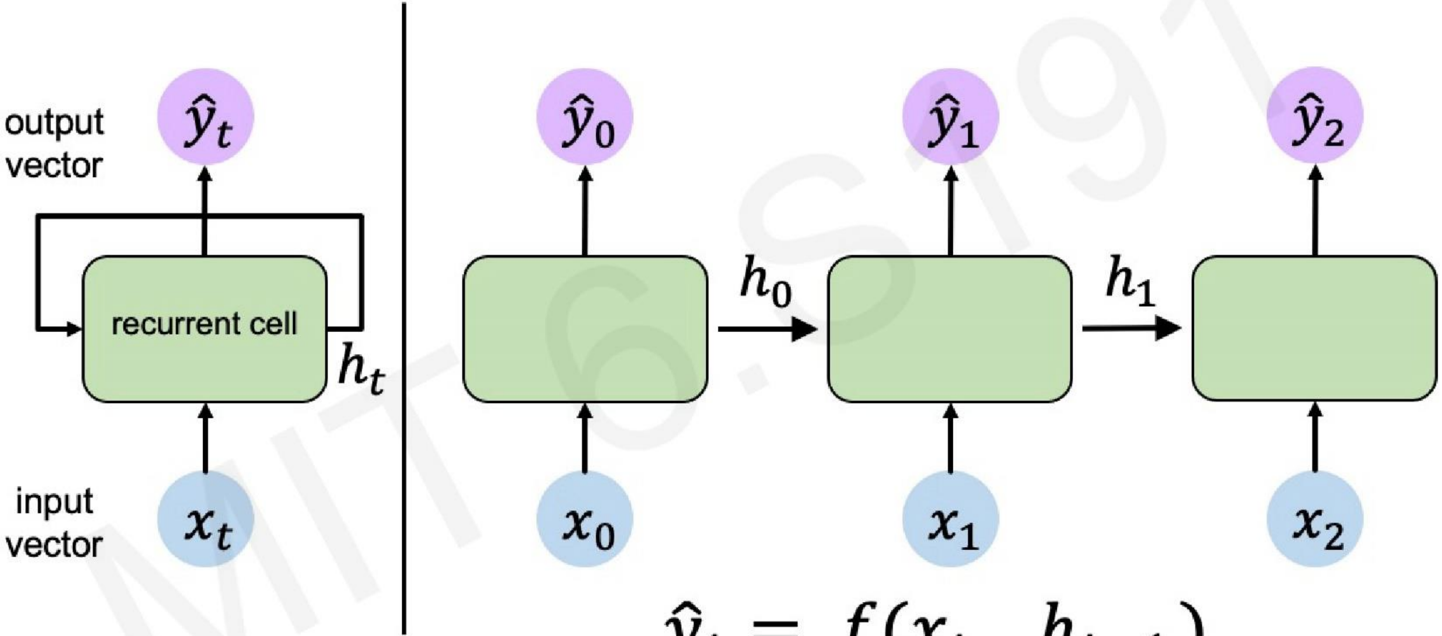
$$\hat{y}_t = f(x_t)$$

Neurons with Recurrence



$$\underbrace{\hat{y}_t}_{\text{output}} = f(\underbrace{x_t}_{\text{input}}, \underbrace{h_{t-1}}_{\text{past memory}})$$

Neurons with Recurrence



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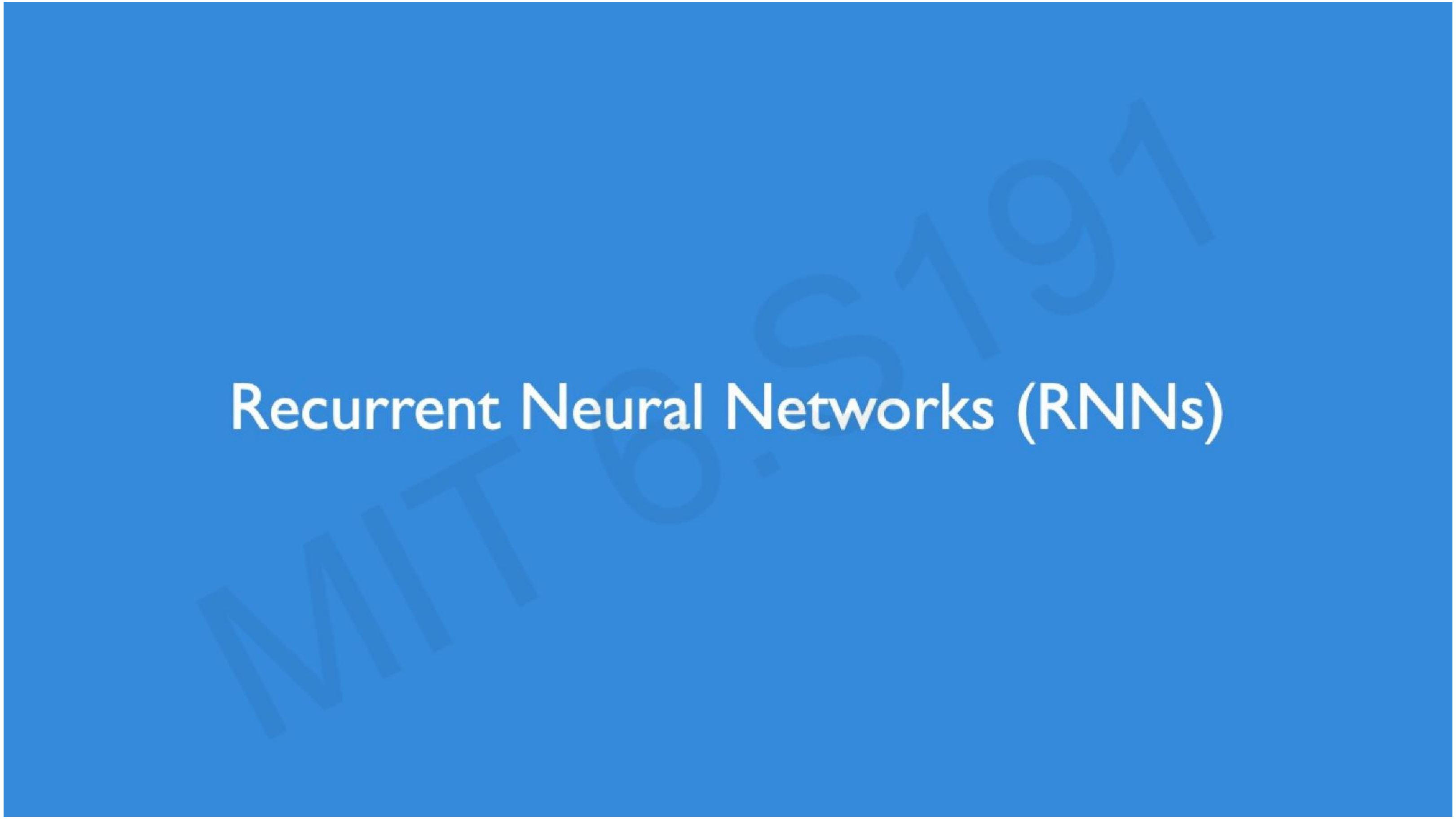
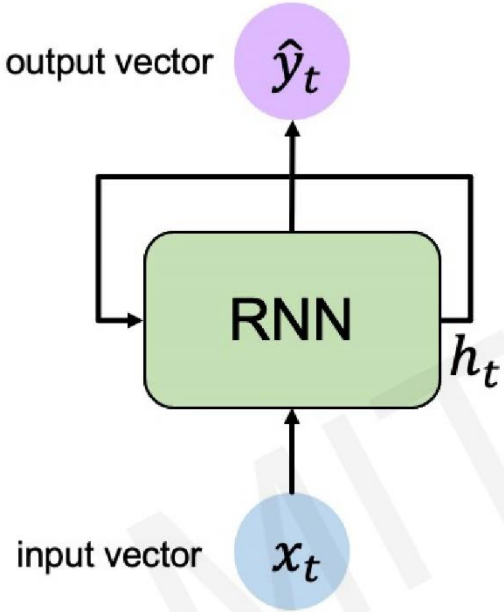

Recurrent Neural Networks (RNNs)
MIT 6.S191

Recurrent Neural Networks (RNNs)



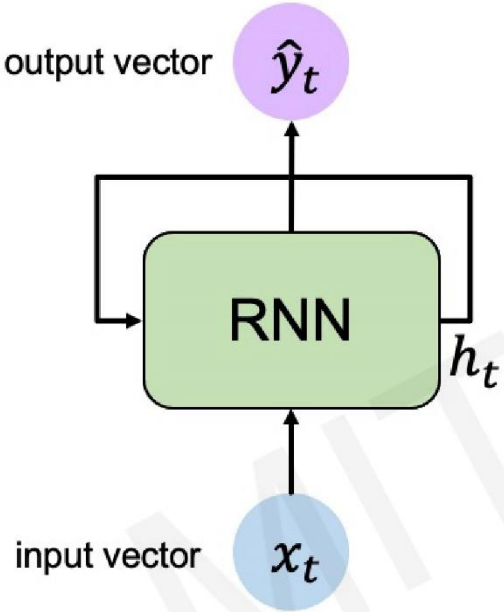
Apply a **recurrence relation** at every time step to process a sequence:

$$\underbrace{\boxed{h_t}}_{\text{cell state}} = \underbrace{\boxed{f_W}}_{\text{function with weights W}}(\underbrace{\boxed{x_t}}_{\text{input}}, \underbrace{\boxed{h_{t-1}}}_{\text{old state}})$$

Note: the same function and set of parameters are used at every time step

RNNs have a **state**, h_t, that is updated **at each time step** as a sequence is processed

RNN State Update and Output



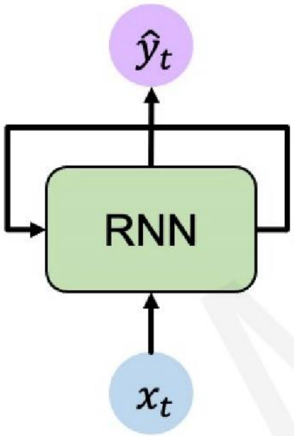
Update Hidden State

$$h_t = \tanh(\boldsymbol{W}_{hh}^T h_{t-1} + \boldsymbol{W}_{xh}^T x_t)$$

Input Vector

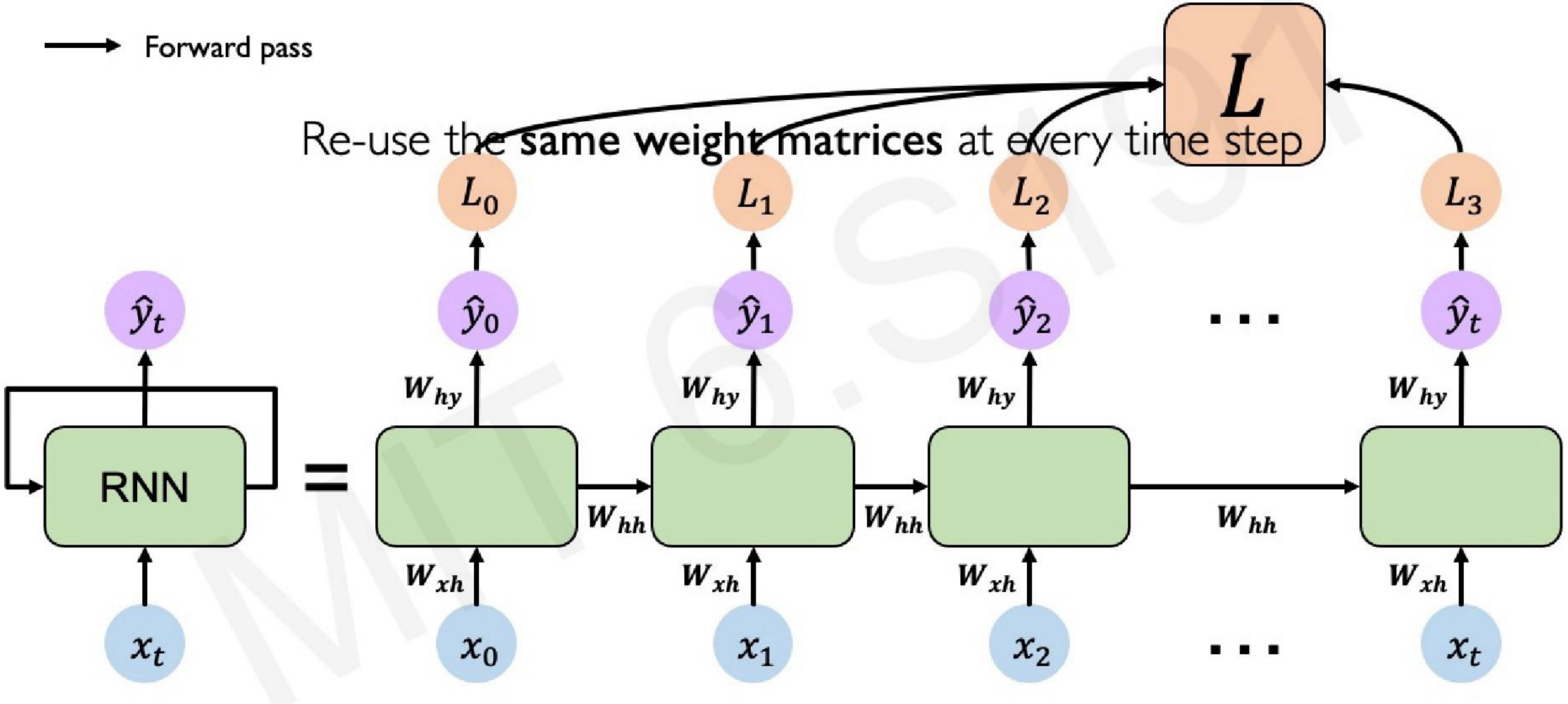
$$x_t$$

RNNs: Computational Graph Across Time



= Represent as computational graph unrolled across time

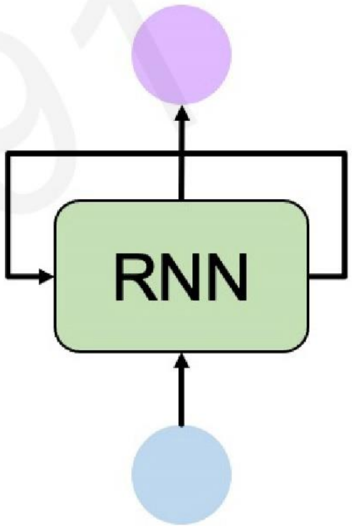
RNNs: Computational Graph Across Time



Sequence Modeling: Design Criteria

To model sequences, we need to:

1. Handle **variable-length** sequences
2. Track **long-term** dependencies
3. Maintain information about **order**
4. **Share parameters** across the sequence



Recurrent Neural Networks (RNNs) meet these sequence modeling design criteria

A Sequence Modeling Problem: Predict the Next Word

A Sequence Modeling Problem: Predict the Next Word

"This morning I took my cat for a walk."

A Sequence Modeling Problem: Predict the Next Word

"This morning I took my cat for a walk."

given these words

A Sequence Modeling Problem: Predict the Next Word

"This morning I took my cat for a walk."

given these words

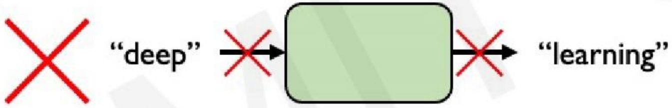
predict the next word

A Sequence Modeling Problem: Predict the Next Word

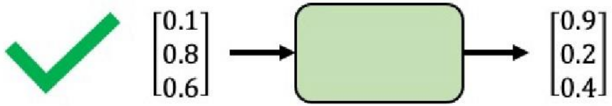
"This morning I took my cat for a walk."

given these words — predict the next word

Representing Language to a Neural Network



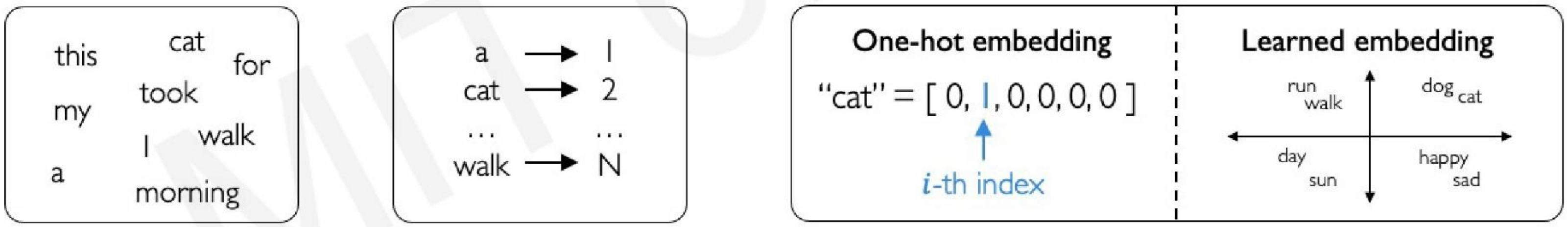
Neural networks cannot interpret words



Neural networks require numerical inputs

Encoding Language for a Neural Network

Embedding: transform indexes into a vector of fixed size.



1. Vocabulary: Corpus of words

2. Indexing: Word to index

3. Embedding: Index to fixed-sized vector

Handle Variable Sequence Lengths

The food was great

vs.

We visited a restaurant for lunch

vs.

We were hungry but cleaned the house before eating

Model Long-Term Dependencies

“**France** is where I grew up, but I now live in Boston. I speak fluent ___.”

We need information from **the distant past** to accurately predict the correct word.

Capture Differences in Sequence Order

The food was good, not bad at all.

vs.

The food was bad, not good at all.

Sequence Modeling: Design Criteria

To model sequences, we need to:

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3. Maintain information about **order**
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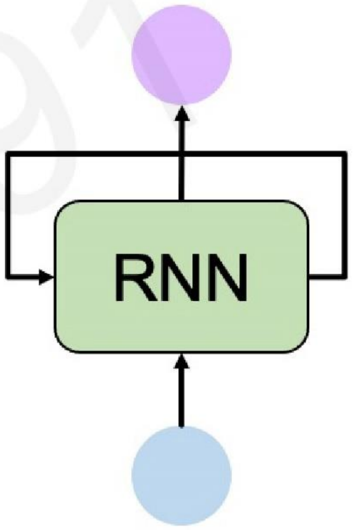
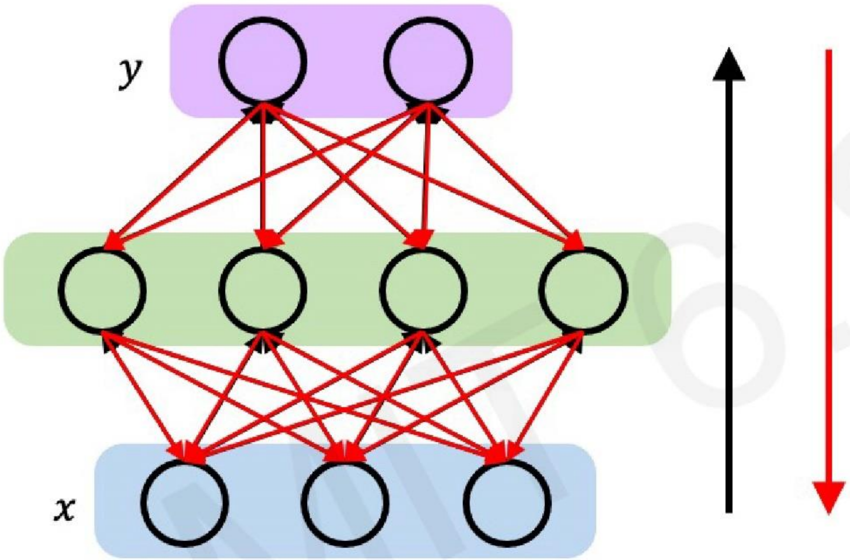


Recurrent Neural Networks (RNNs) meet these sequence modeling design criteria

Backpropagation Through Time (BPTT)

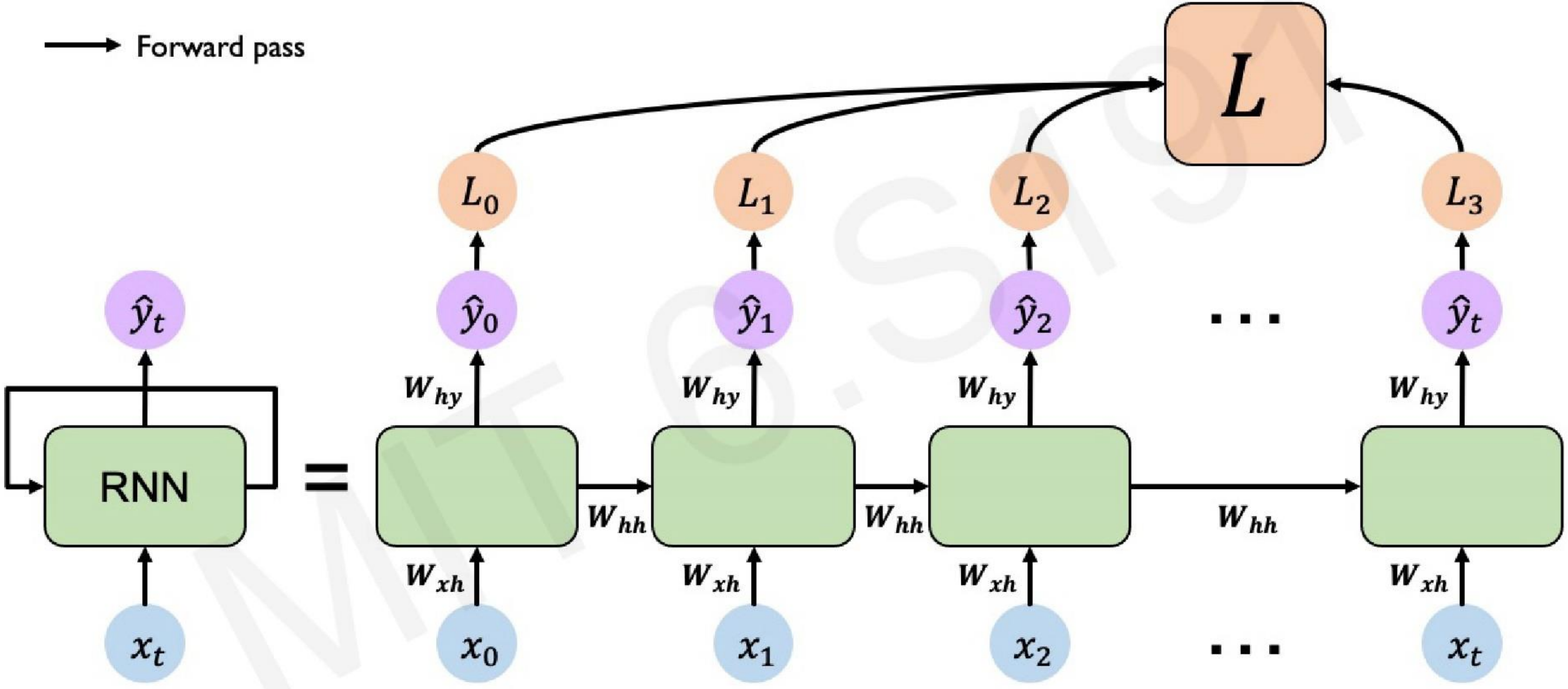
Recall: Backpropagation in Feed Forward Models



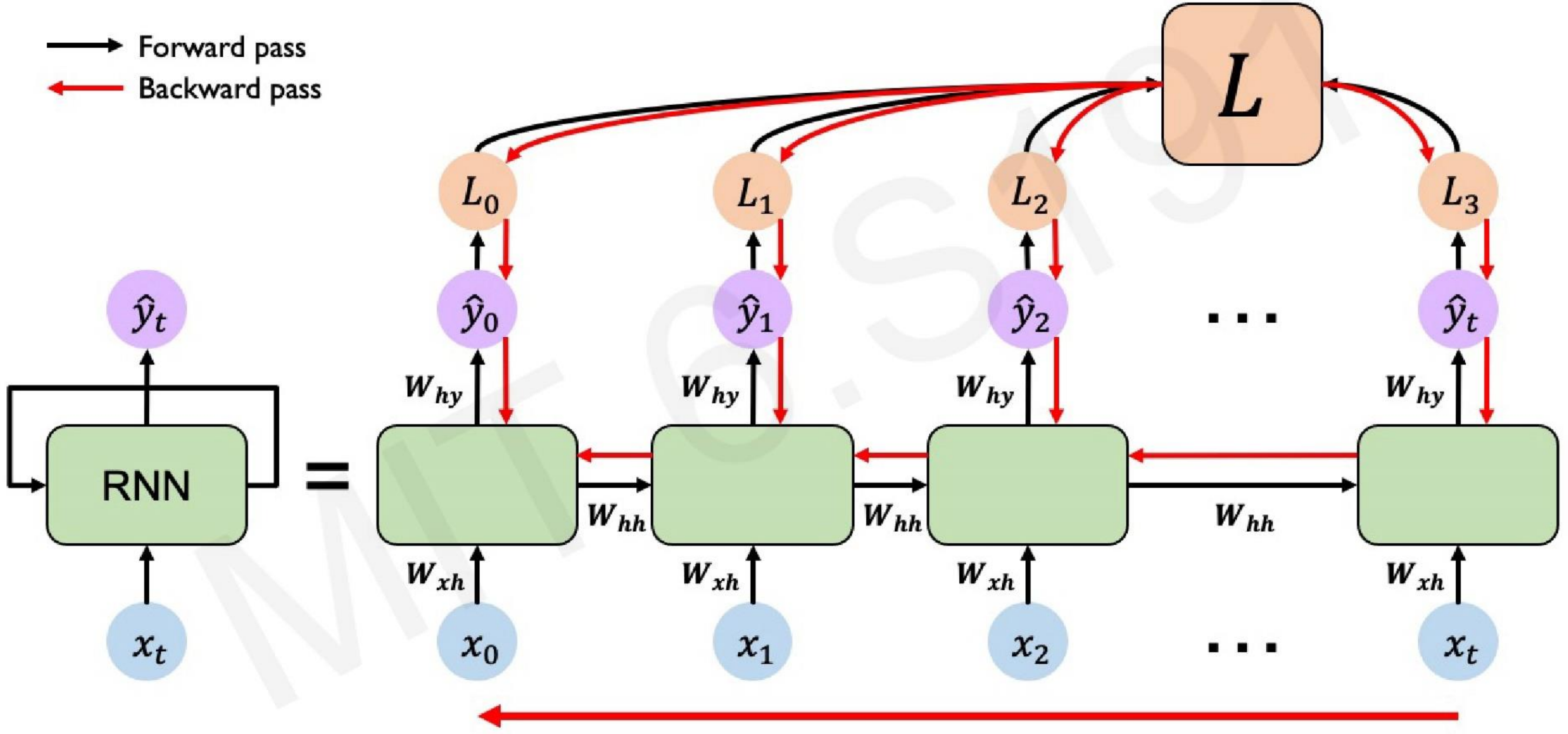
Backpropagation algorithm:

1. Take the derivative (gradient) of the loss with respect to each parameter
2. Shift parameters in order to minimize loss

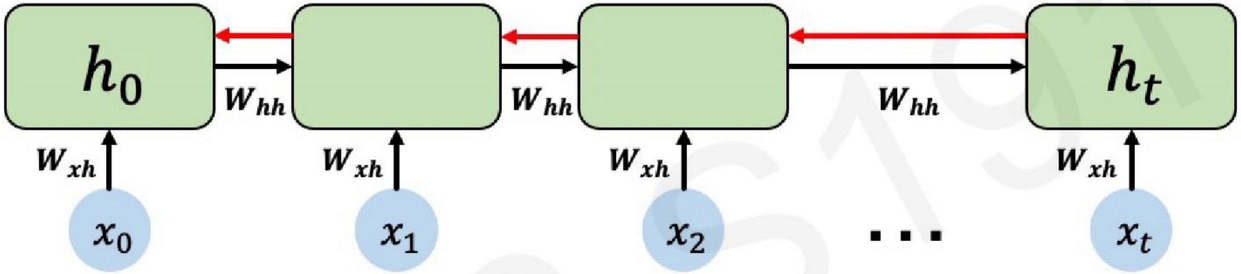
RNNs: Backpropagation Through Time



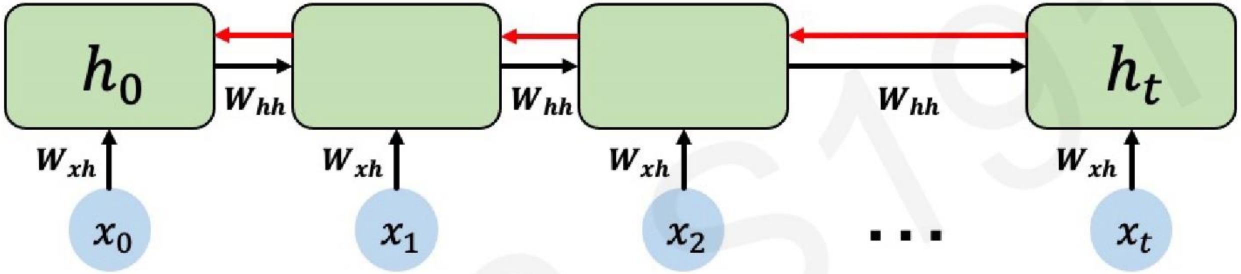
RNNs: Backpropagation Through Time



Standard RNN Gradient Flow

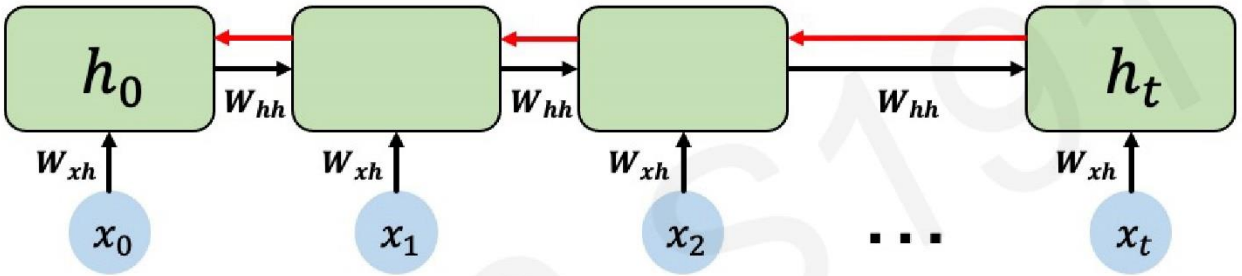


Standard RNN Gradient Flow



Computing the gradient wrt h_0 involves **many factors of W_{hh}** + **repeated gradient computation!**

Standard RNN Gradient Flow: Exploding Gradients

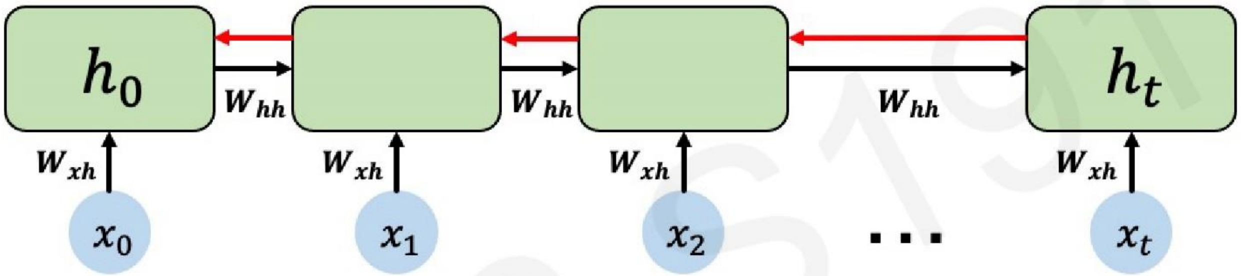


Computing the gradient wrt h_0 involves **many factors of W_{hh} + repeated gradient computation!**

Many values > 1:
exploding gradients

Gradient clipping to
scale big gradients

Standard RNN Gradient Flow: Vanishing Gradients



Computing the gradient wrt h_0 involves **many factors of W_{hh} + repeated gradient computation!**

Many values > 1:
exploding gradients

Gradient clipping to scale big gradients

Many values < 1:
vanishing gradients

1. Activation function
2. Weight initialization
3. Network architecture

The Problem of Long-Term Dependencies

Why are vanishing gradients a problem?

Multiply many **small numbers** together

↓

Errors due to further back time steps
have smaller and smaller gradients

↓

Bias parameters to capture short-term
dependencies

The Problem of Long-Term Dependencies

"The clouds are in the ___"

Why are vanishing gradients a problem?

Multiply many **small numbers** together

↓

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The Problem of Long-Term Dependencies

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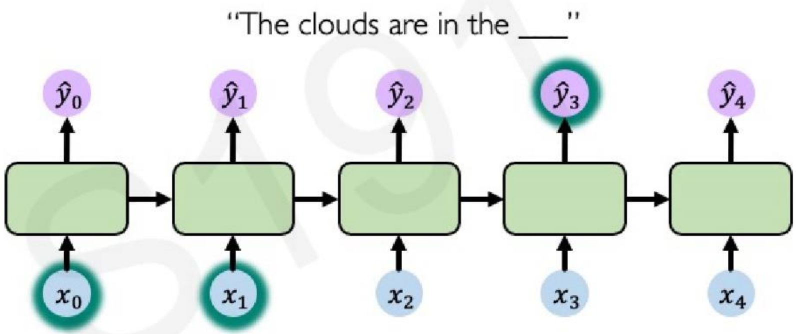

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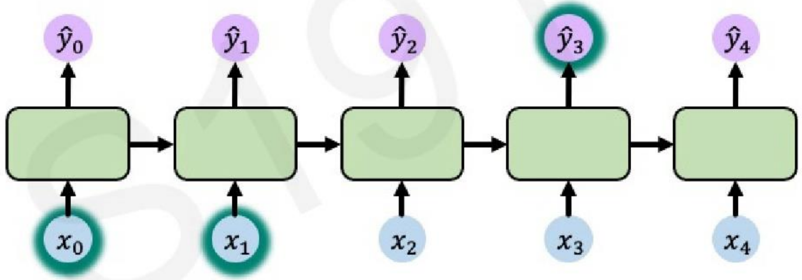
"The clouds are in the ___"



"I grew up in France, … and I speak fluent___ "

The Problem of Long-Term Dependencies

Why are vanishing gradients a problem?

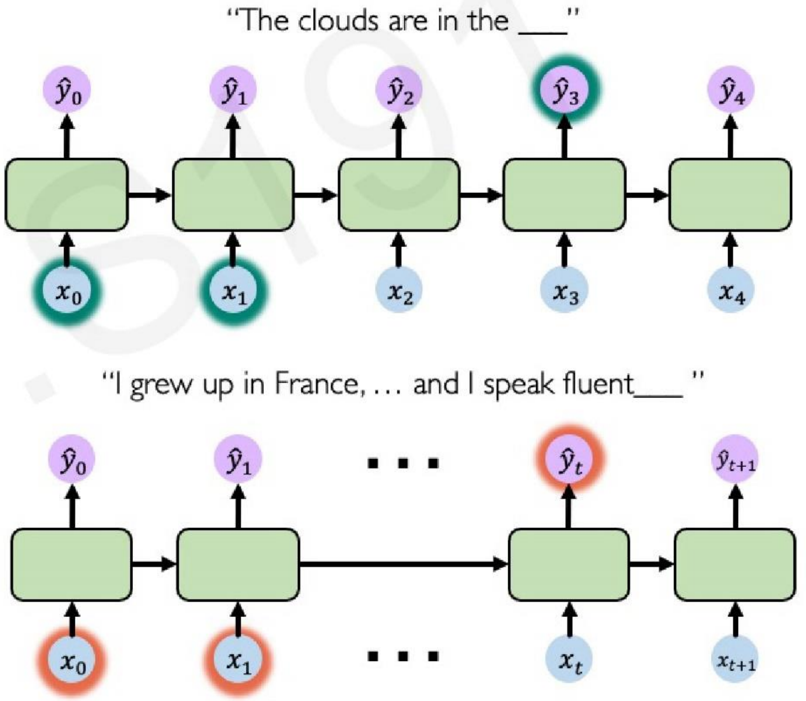
Multiply many **small numbers** together

↓

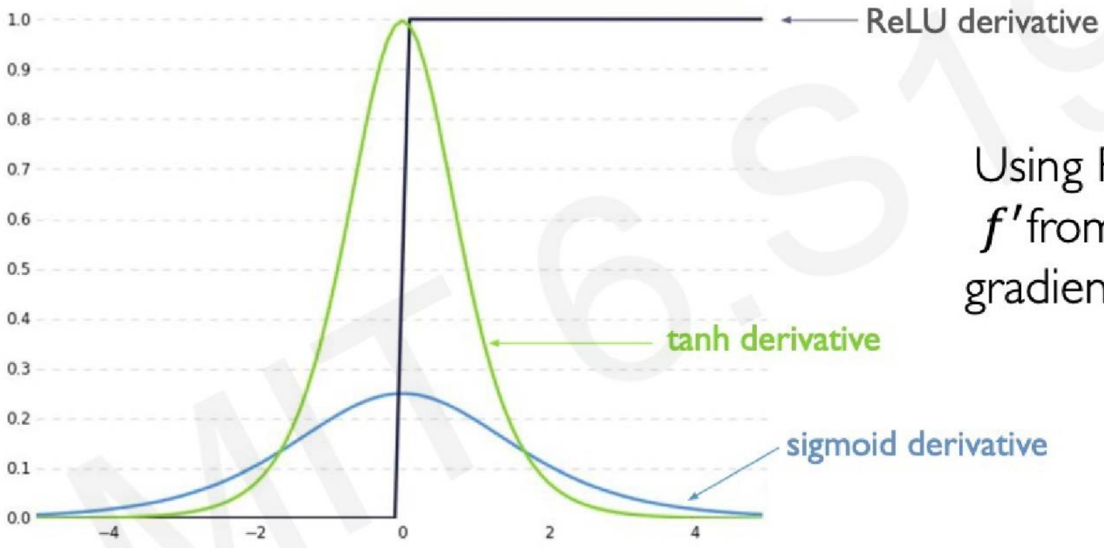
Errors due to further back time steps have smaller and smaller gradients

↓

Bias parameters to capture short-term dependencies



Trick #1: Activation Functions



Using ReLU prevents f' from shrinking the gradients when $x > 0$

Trick #2: Parameter Initialization

Initialize **weights** to identity matrix

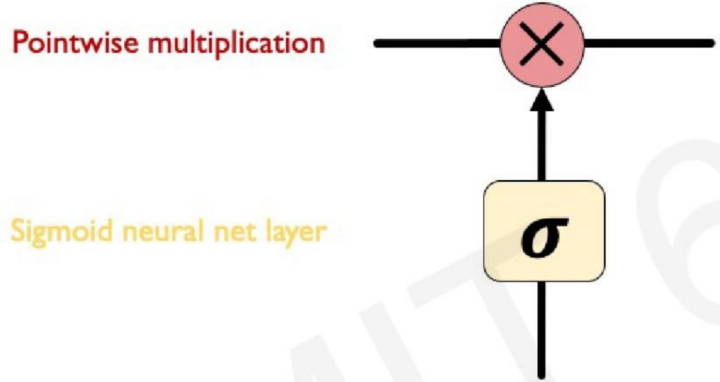
Initialize **biases** to zero

$$I_n = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}$$

This helps prevent the weights from shrinking to zero.

Trick #3: Gated Cells

Idea: use **gates** to selectively **add** or **remove** information within **each recurrent unit with**



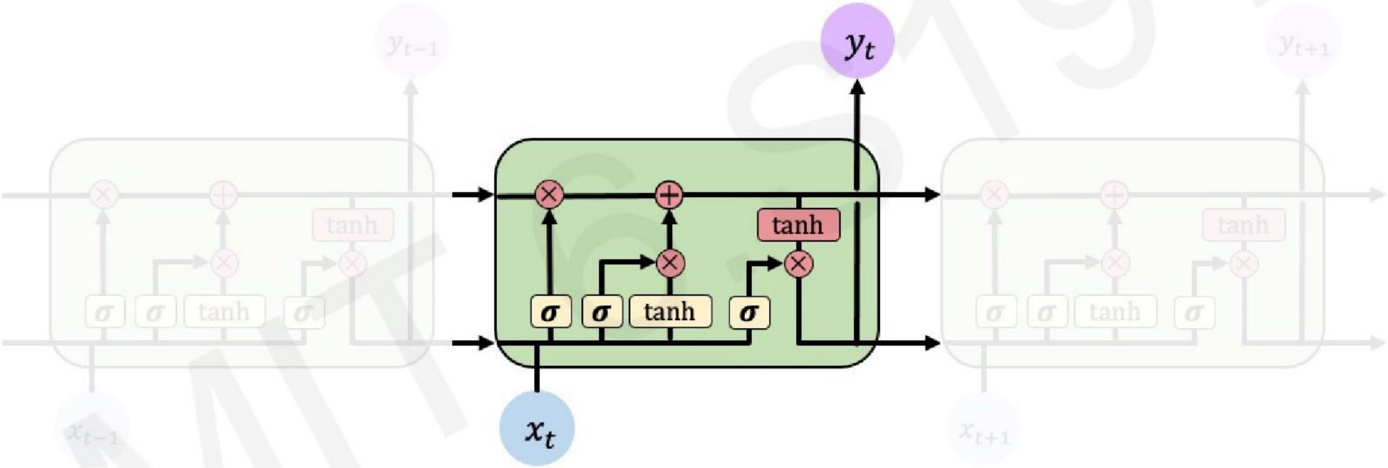
Gates optionally let information through the cell

Long Short Term Memory (LSTMs) networks rely on a gated cell to track information throughout many time steps.

Long Short Term Memory (LSTMs)

Gated LSTM cells **control information flow:**

1) Forget 2) Store 3) Update 4) Output



LSTM cells are able to track information throughout many timesteps

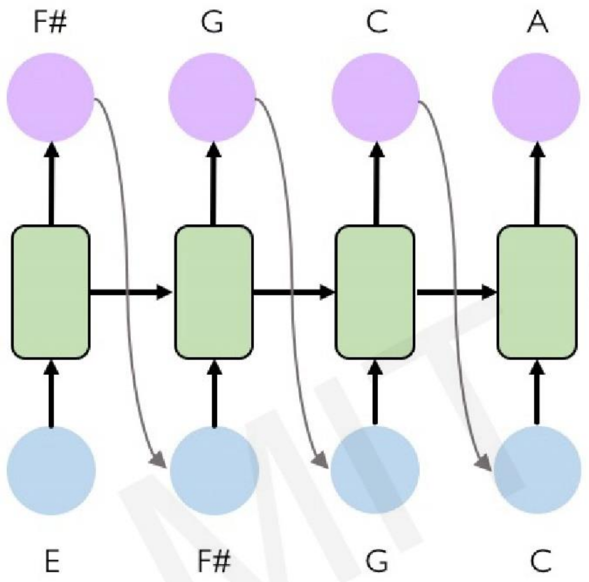
```
tf.keras.layers.LSTM(num_units)
```

LSTMs: Key Concepts

1. Maintain a **cell state**
2. Use **gates** to control the **flow of information**
    - **Forget** gate gets rid of irrelevant information
    - **Store** relevant information from current input
    - Selectively **update** cell state
    - **Output** gate returns a filtered version of the cell state
3. Backpropagation through time with partially **uninterrupted gradient flow**

RNN Applications & Limitations

Example Task: Music Generation

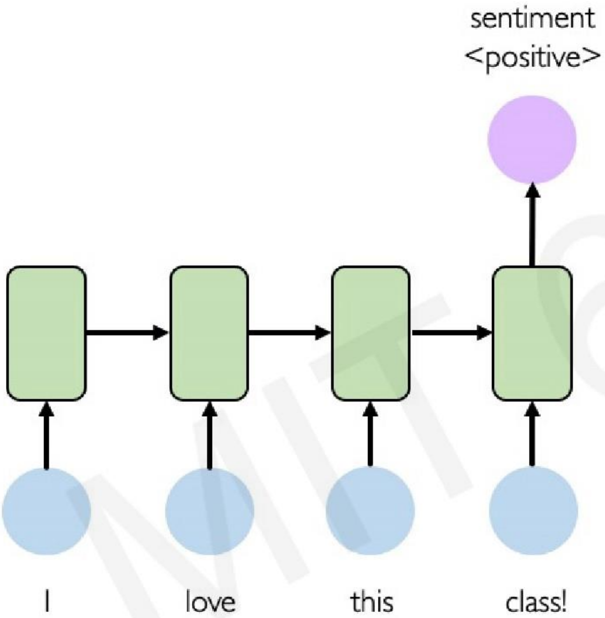


Input: sheet music

Output: next character in sheet music

Software Lab!

Example Task: Sentiment Classification

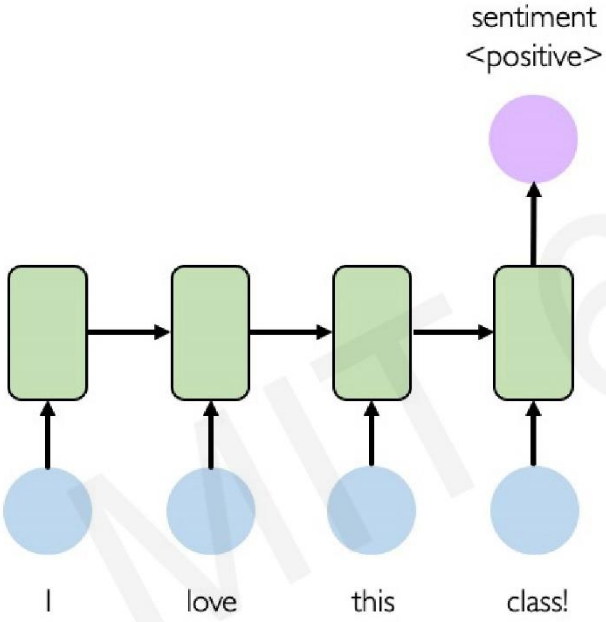


Input: sequence of words

Output: probability of having positive sentiment

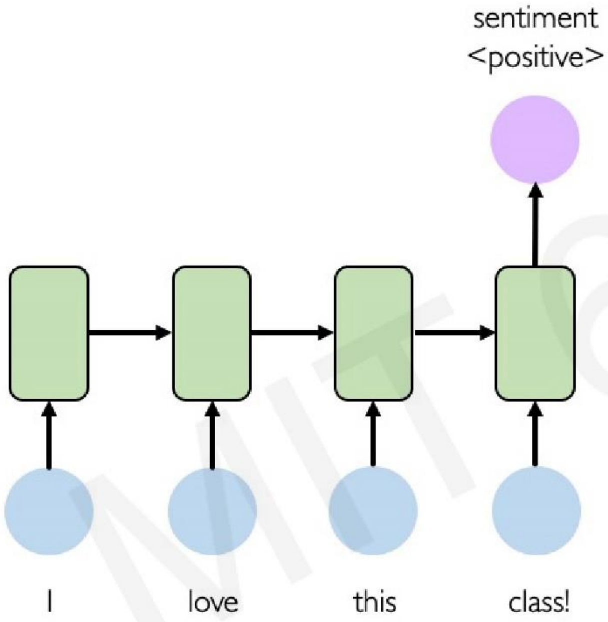
```
loss = tf.nn.softmax_cross_entropy_with_logits(y, predicted)
```

Example Task: Sentiment Classification



Tweet sentiment classification

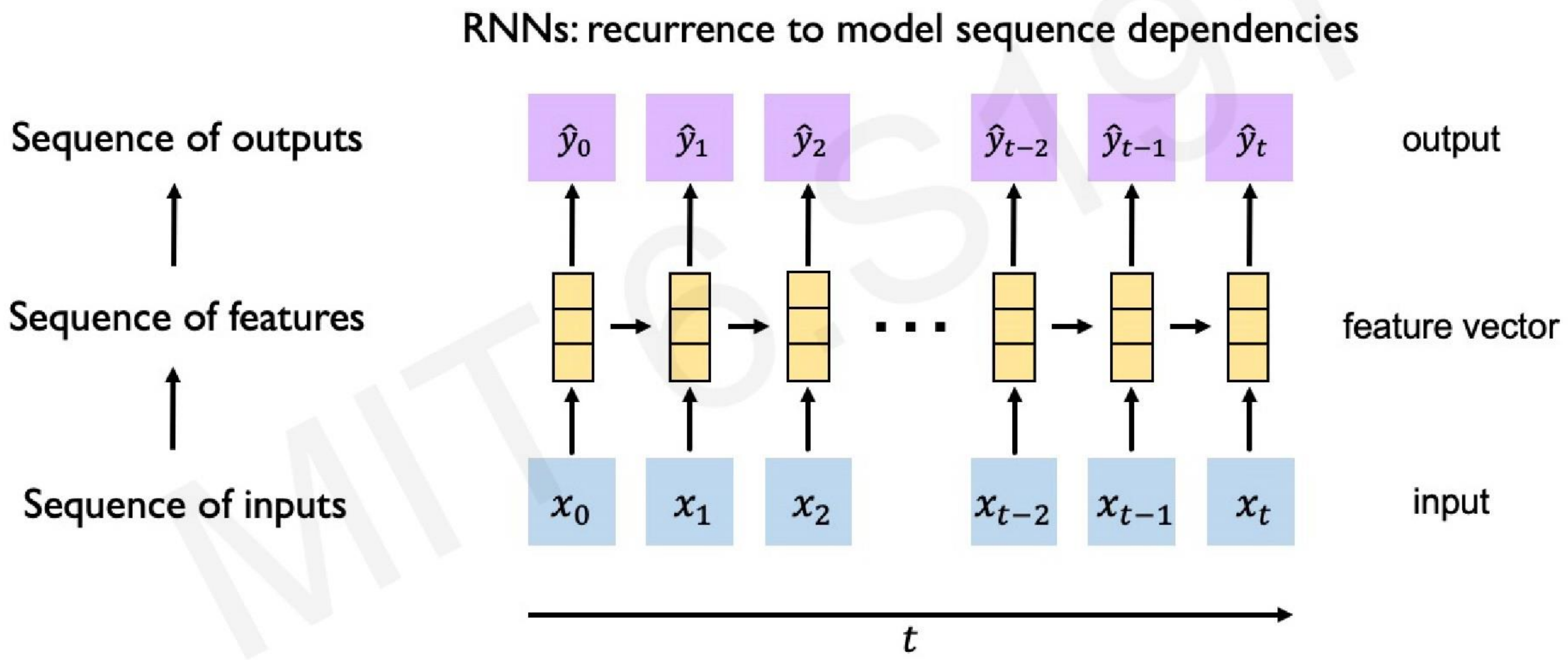
Limitations of Recurrent Models



Limitations of RNNs

- Encoding bottleneck
- Slow, no parallelization
- Not long memory

Goal of Sequence Modeling

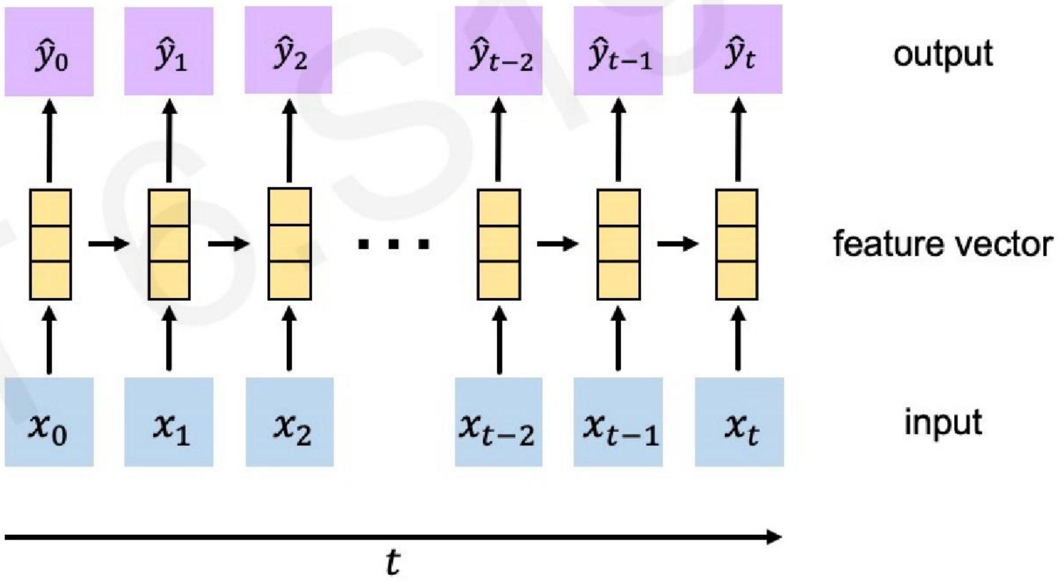


Goal of Sequence Modeling

Limitations of RNNs

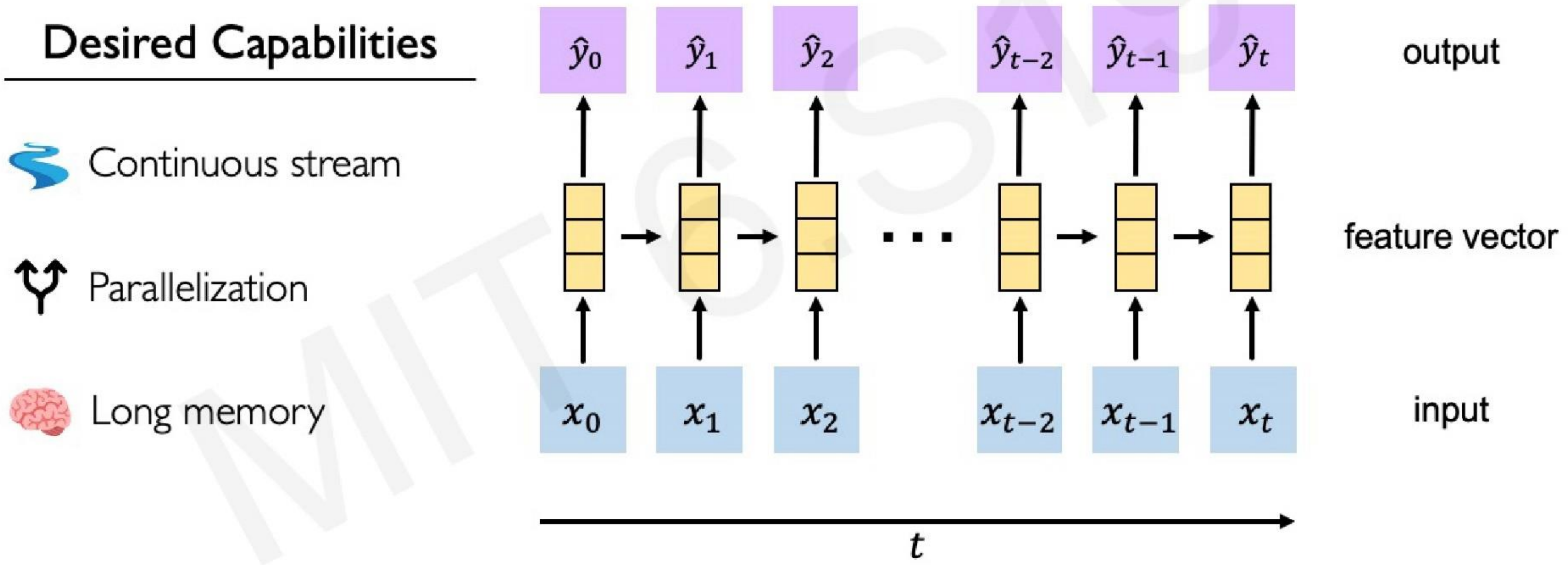
- Encoding bottleneck
- Slow, no parallelization
- Not long memory

RNNs: recurrence to model sequence dependencies



Goal of Sequence Modeling

Can we eliminate the need for recurrence entirely?

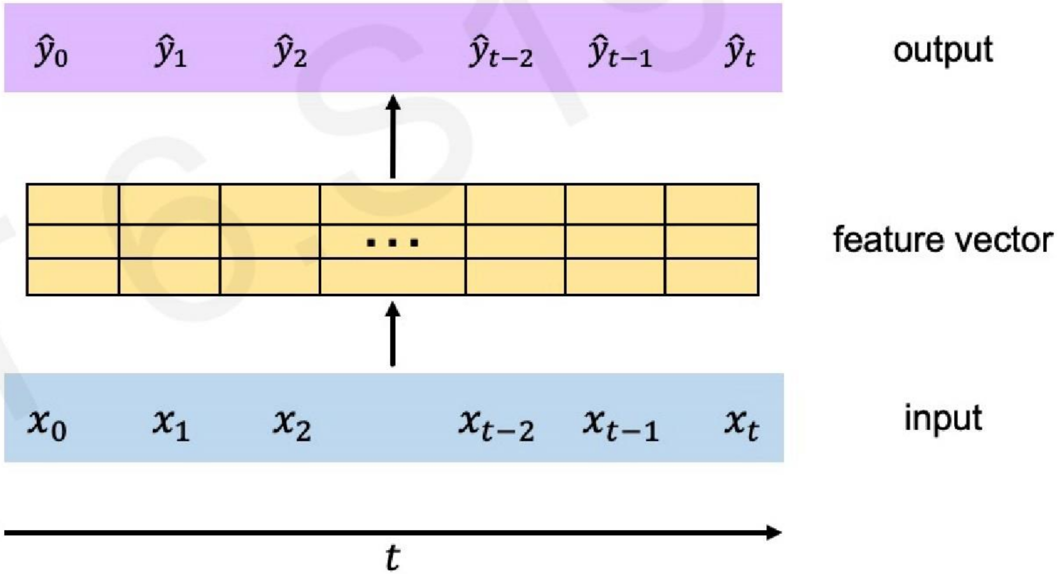


Goal of Sequence Modeling

Can we eliminate the need for recurrence entirely?

Desired Capabilities

- Continuous stream
- Parallelization
- Long memory



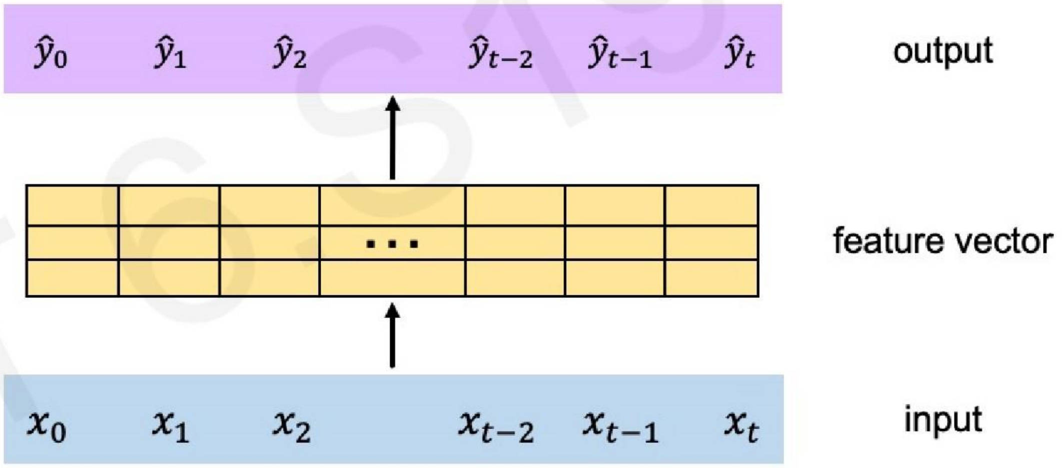
Goal of Sequence Modeling

Idea 1: Feed everything into dense network

- ✓ No recurrence
- ✗ Not scalable
- ✗ No order
- ✗ No long memory

Idea: Identify and attend to what's important

Can we eliminate the need for recurrence entirely?



人工智能基本理论

Intuition Behind Self-Attention

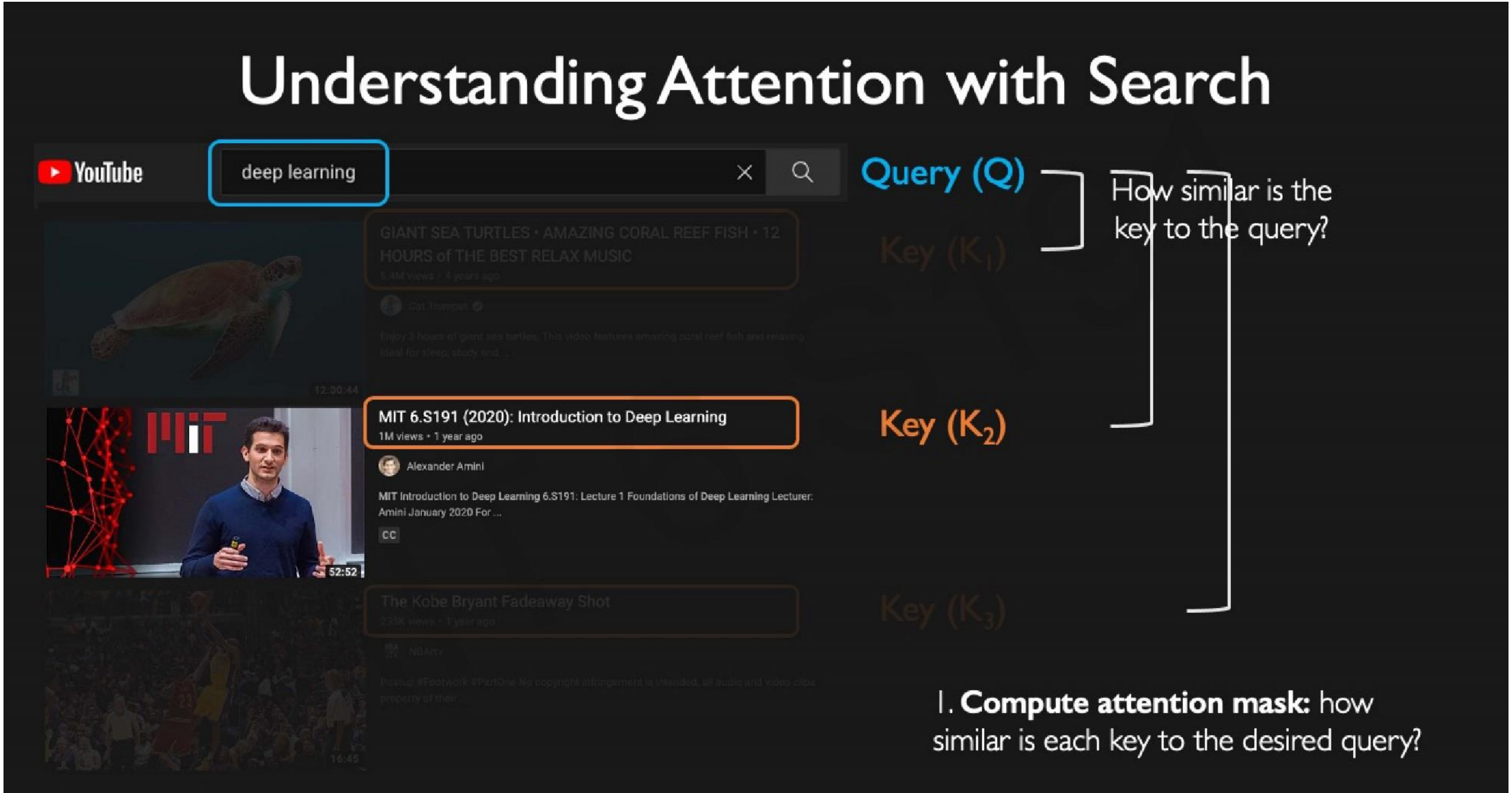
Attending to the most important parts of an input.

1. $\boxed{\text{Identify which parts to attend to}}$
2. Extract the features with high attention

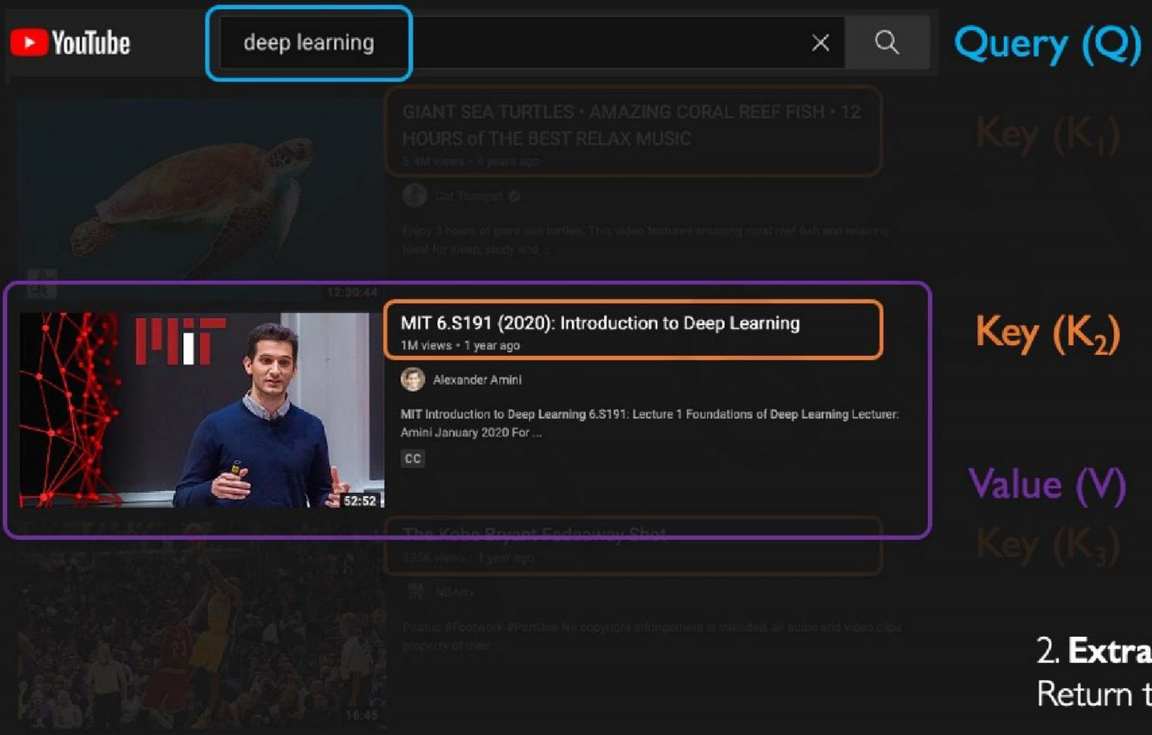
Similar to a search problem!

A Simple Example: Search

人工智能基本理论


Understanding Attention with Search
YouTube
deep learning
Query (Q)
How similar is the key to the query?
GIANT SEA TURTLES • AMAZING CORAL REEF FISH • 12 HOURS of THE BEST RELAX MUSIC
Key (K_1)
MIT 6.S191 (2020): Introduction to Deep Learning
1M views • 1 year ago
Alexander Amini
MIT Introduction to Deep Learning 6.S191: Lecture 1 Foundations of Deep Learning Lecturer: Amini January 2020 For ...
CC
52:52
Key (K_2)
The Kobe Bryant Fadeaway Shot
Key (K_3)
1. **Compute attention mask:** how similar is each key to the desired query?

Understanding Attention with Search



2. **Extract values based on attention:**
Return the values highest attention

Learning Self-Attention with Neural Networks

Goal: identify and attend to most important features in input.

1. Encode **position** information
2. Extract **query, key, value** for search
3. Compute **attention weighting**
4. Extract **features with high attention**

Data is fed in all at once! Need to encode position information to understand order.

Learning Self-Attention with Neural Networks

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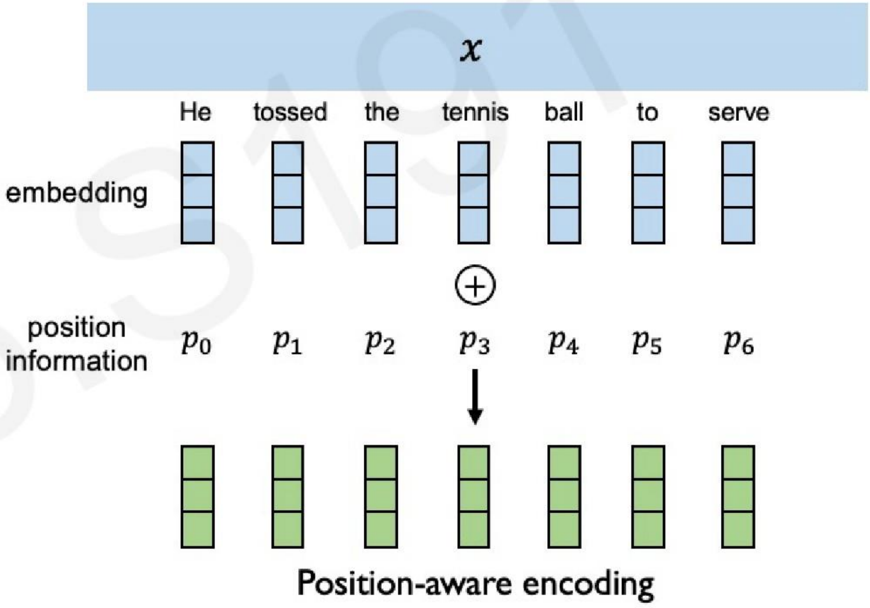

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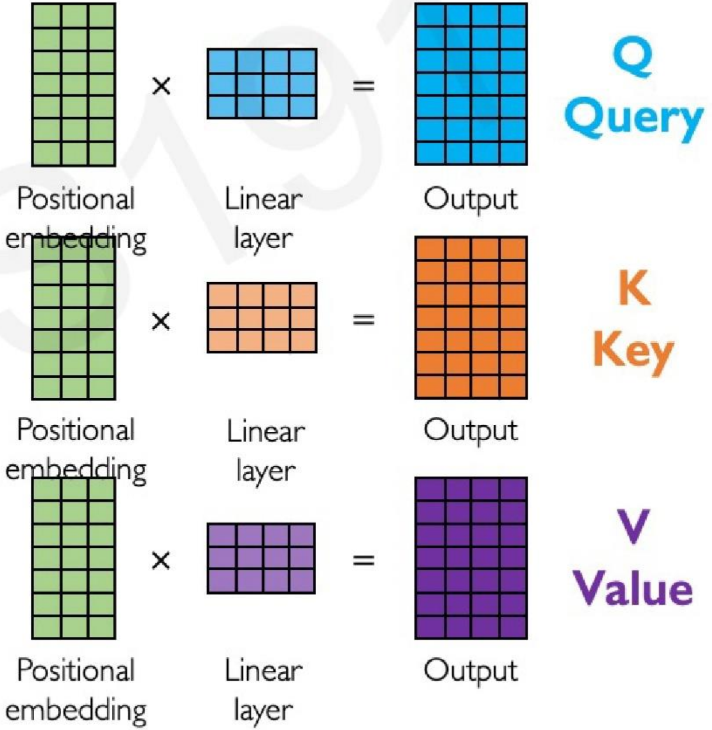

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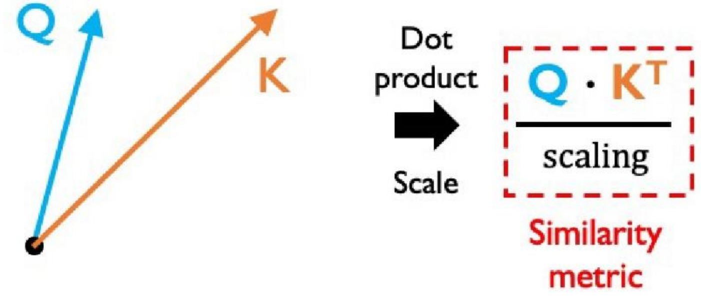
4. Extract **features with high attention**

Attention score: compute pairwise similarity between each query and key

How to compute similarity between two sets of features?



Also known as the "cosine similarity"

Learning Self-Attention with Neural Networks

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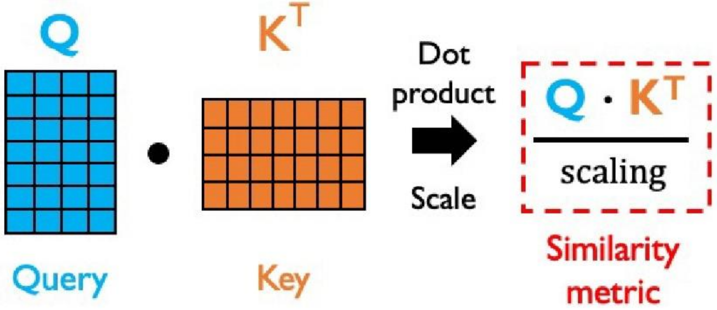

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Learning Self-Attention with Neural Networks

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2. Extract **query, key, value** for search
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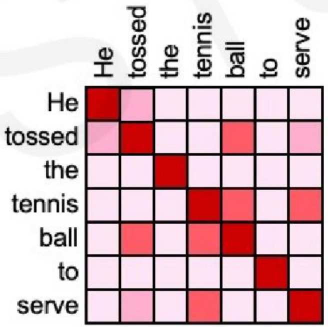
4. Extract **features with high attention**

Attention weighting: where to attend to!
How similar is the key to the query?



$$softmax\left(\frac{Q \cdot K^T}{scaling}\right)$$

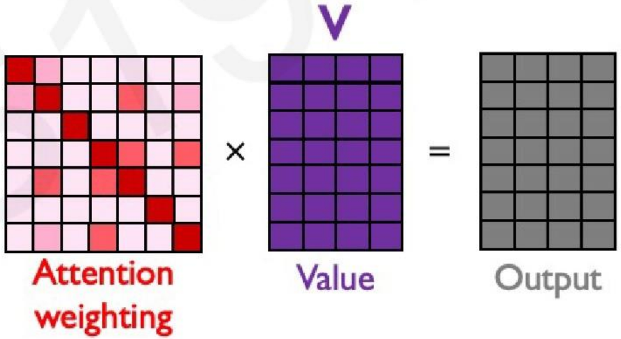
Attention weighting

Learning Self-Attention with Neural Networks

Goal: identify and attend to most important features in input.

1. Encode **position** information
2. Extract **query, key, value** for search
3. Compute **attention weighting**
4. Extract **features with high attention**

Last step: self-attend to extract features



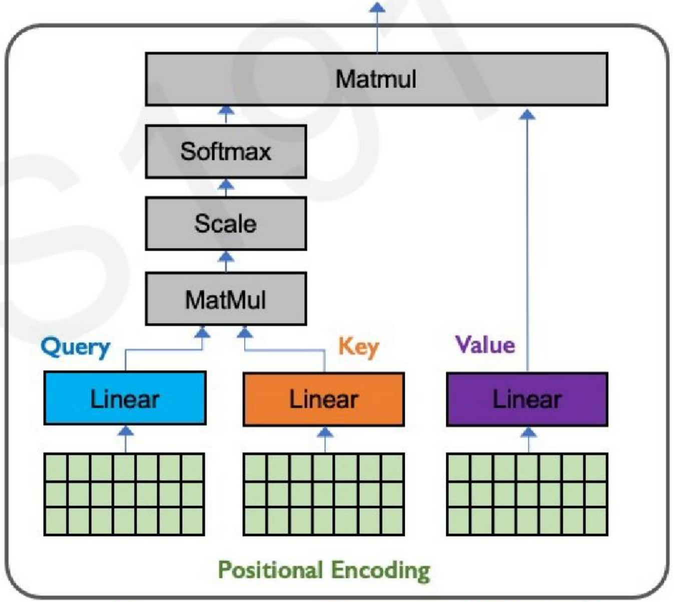
$$\underbrace{softmax\left(\frac{Q \cdot K^T}{scaling}\right)} \cdot \underbrace{V} = \underbrace{A(Q,K,V)}$$

Learning Self-Attention with Neural Networks

Goal: identify and attend to most important features in input.

1. Encode **position** information
2. Extract **query, key, value** for search
3. Compute **attention weighting**
4. Extract **features with high attention**

These operations form a self-attention head that can plug into a larger network.
Each head attends to a different part of input.



$$softmax\left(\frac{\boxed{Q \cdot K^T}}{scaling}\right) \cdot V$$

Applying Multiple Self-Attention Heads

Attention weighting

×

Value

=

Output

Output of attention head 1

Output of attention head 2

Output of attention head 3

Self-Attention Applied

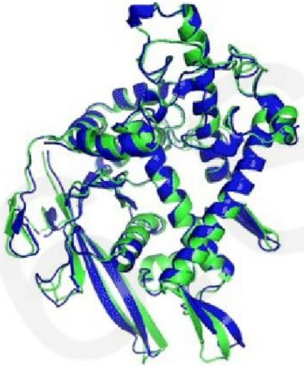
Language Processing

An armchair in the shape of an avocado

BERT, GPT-3

Devlin et al., *NAACL* 2019
Brown et al., *NeurIPS* 2020

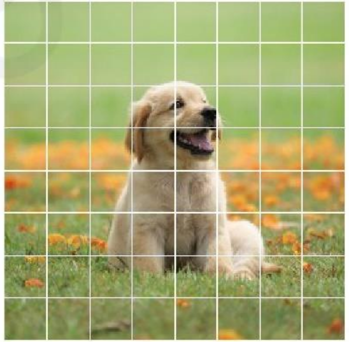
Biological Sequences



AlphaFold2

Jumper et al., *Nature* 2021

Computer Vision

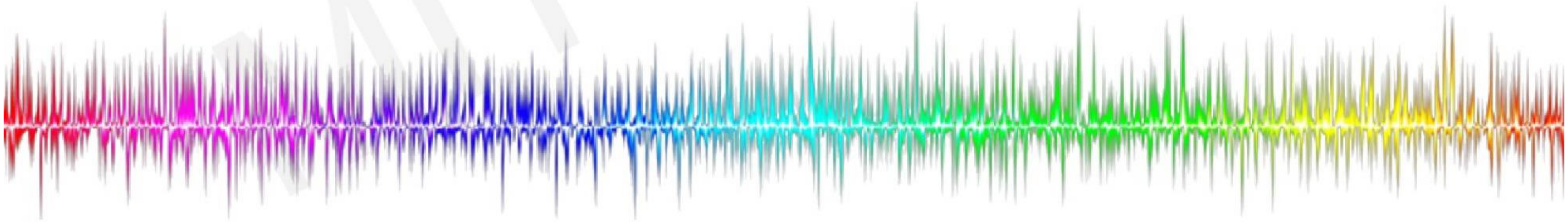


Vision Transformers

Dosovitskiy et al., *ICLR* 2020

Deep Learning for Sequence Modeling: Summary

1. RNNs are well suited for **sequence modeling** tasks
2. Model sequences via a **recurrence relation**
3. Training RNNs with **backpropagation through time**
4. Models for **music generation**, classification, machine translation, and more
5. Self-attention to model **sequences without recurrence**



提纲

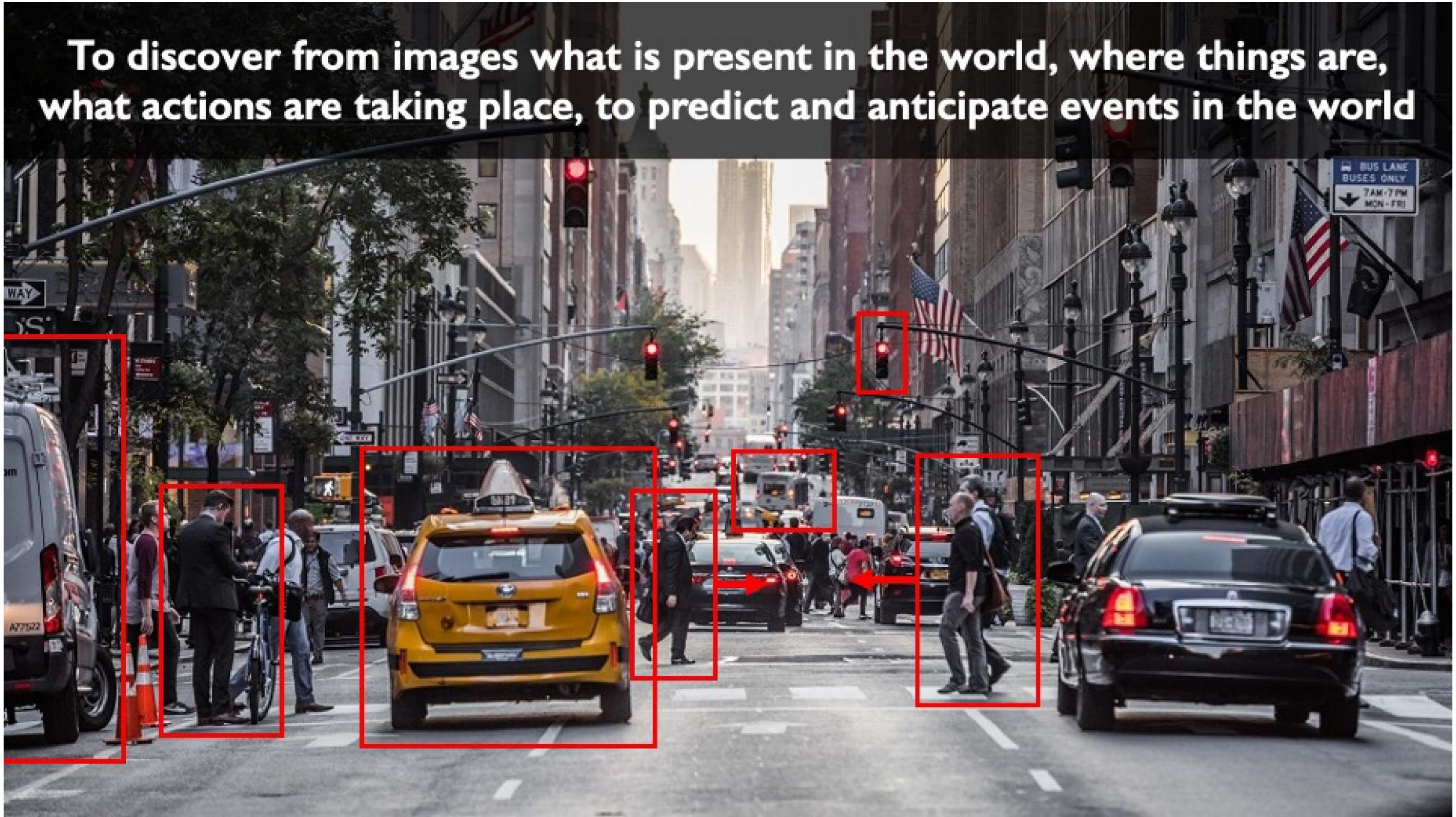
一、前馈网络

二、循环网络

三、卷积网络

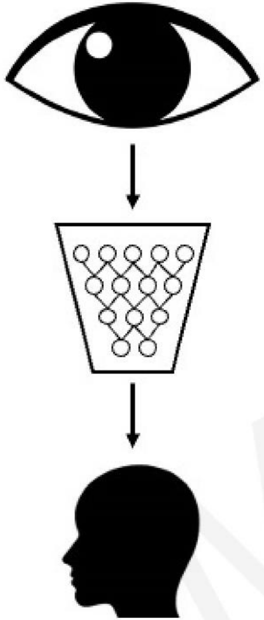
人工智能基本理论

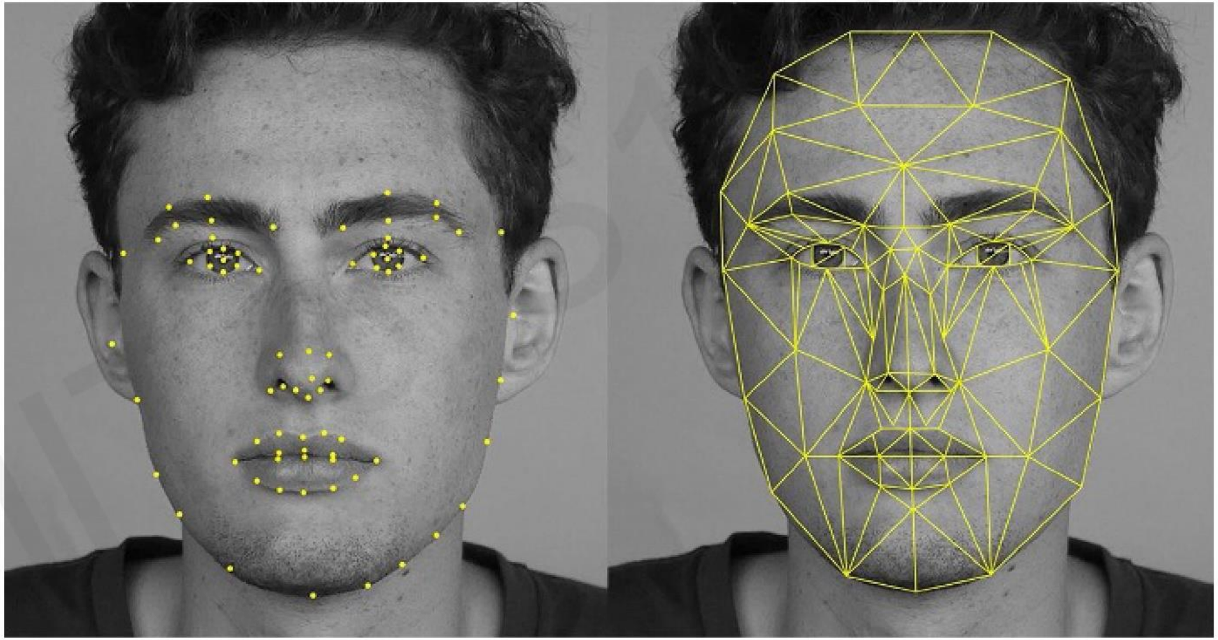
人工智能基本理论



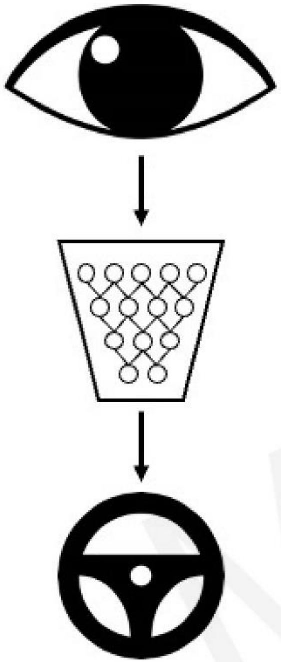
To discover from images what is present in the world, where things are,
what actions are taking place, to predict and anticipate events in the world
BUS LANE
BUSES ONLY
7AM-7PM
MON-FRI
WAY

Impact: Facial Detection & Recognition



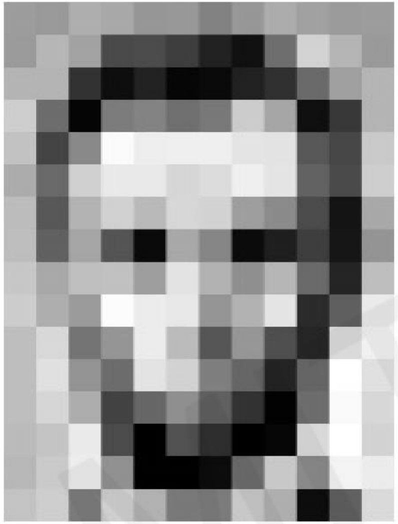


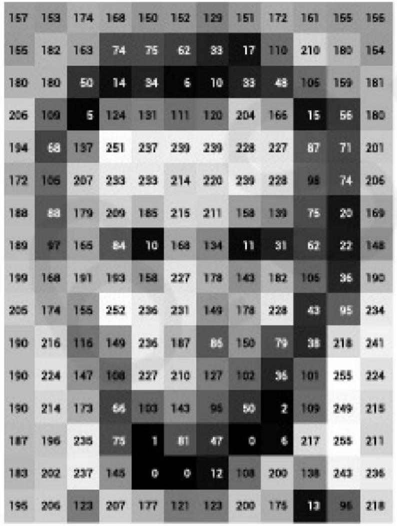
Impact: Self-Driving Cars




What Computers “See”
MIT 6.S191

Images are Numbers



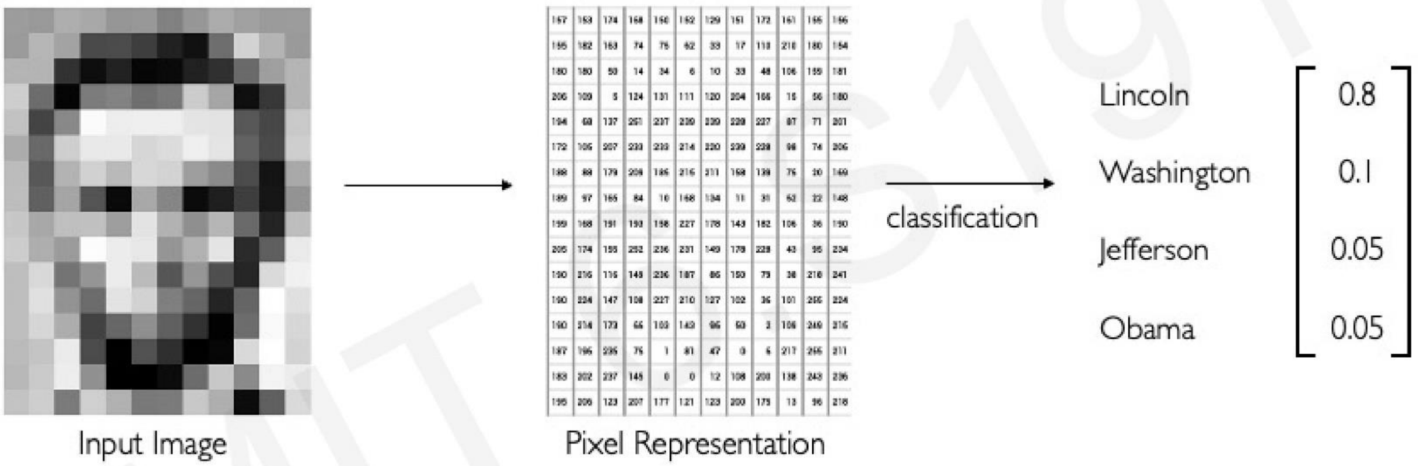


What the computer sees

157	153	174	168	150	152	129	151	172	161	155	156
155	182	163	74	75	62	33	17	110	210	180	154
180	180	50	14	34	6	10	33	48	106	159	181
206	109	5	124	131	111	120	204	166	15	56	180
194	68	137	251	237	239	239	228	227	87	71	201
172	105	207	233	233	214	220	239	228	98	74	206
188	88	179	209	185	215	211	158	139	75	20	169
189	97	165	84	10	168	134	11	31	62	22	148
199	168	191	193	158	227	178	143	182	106	36	190
205	174	155	252	236	231	149	178	228	43	95	234
190	216	116	149	236	187	86	150	79	38	218	241
190	224	147	108	227	210	127	102	36	101	255	224
190	214	173	66	103	143	96	50	2	109	249	215
187	196	235	75	1	81	47	0	6	217	255	211
183	202	237	145	0	0	12	108	200	138	243	236
195	206	123	207	177	121	123	200	175	13	96	218

An image is just a matrix of numbers [0,255]!
i.e., 1080x1080x3 for an RGB image

Tasks in Computer Vision



- **Regression**: output variable takes continuous value
- **Classification**: output variable takes class label. Can produce probability of belonging to a particular class

High Level Feature Detection

Let's identify key features in each image category

Nose,
Eyes,
Mouth

Wheels,
License Plate,
Headlights

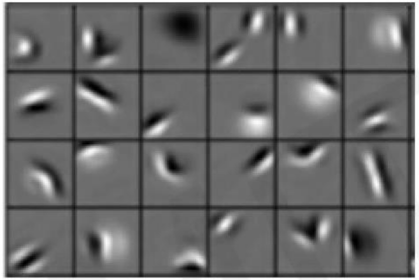
Door,
Windows,
Steps

Manual Feature Extraction

Learning Feature Representations

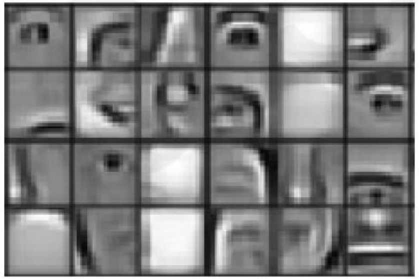
Can we learn a **hierarchy of features** directly from the data instead of hand engineering?

Low level features



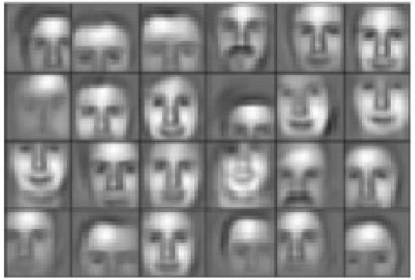
Edges, dark spots

Mid level features



Eyes, ears, nose

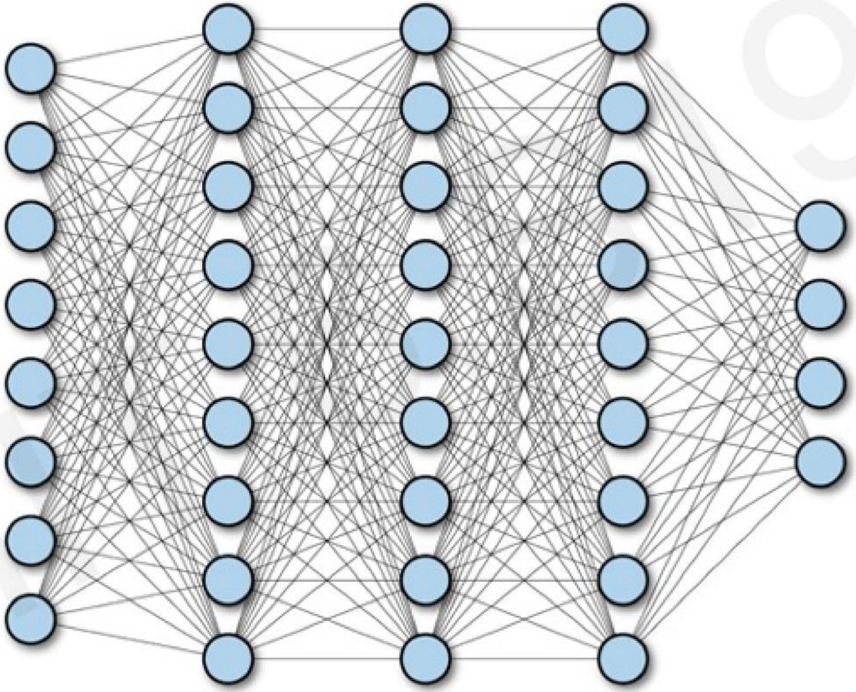
High level features



Facial structure

Learning Visual Features

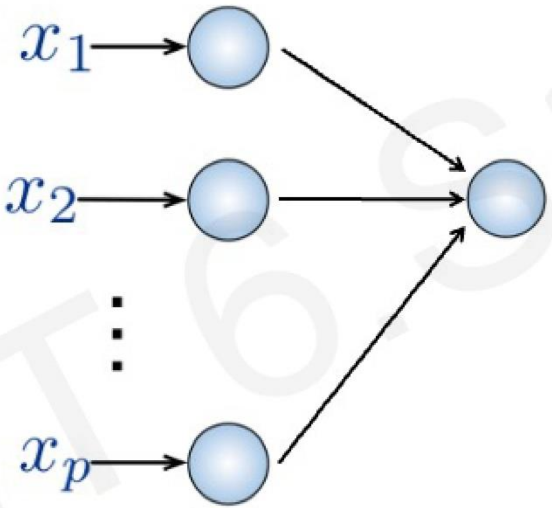
Fully Connected Neural Network



Fully Connected Neural Network

Input:

- 2D image
- Vector of pixel values



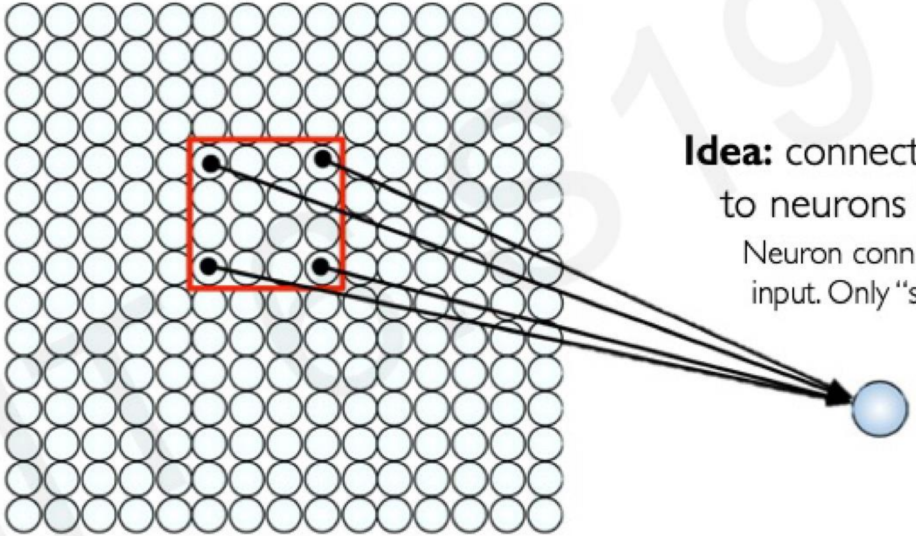
Fully Connected:

- Connect neuron in hidden layer to all neurons in input layer
- No spatial information!
- And many, many parameters!

How can we use **spatial structure** in the input to inform the architecture of the network?

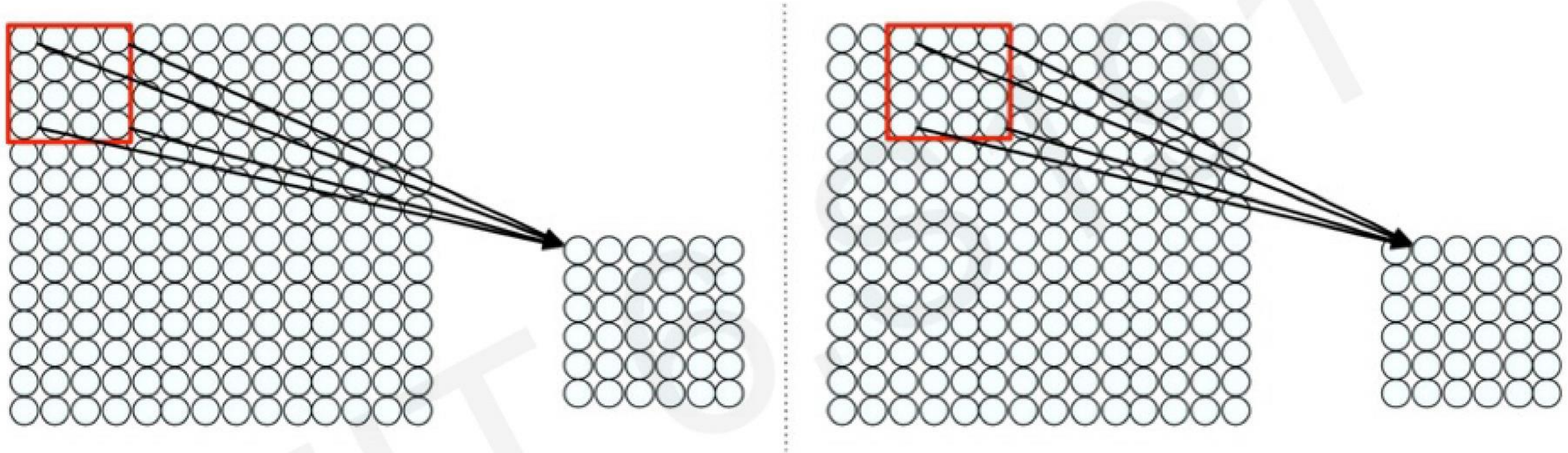
Using Spatial Structure

Input: 2D image.
Array of pixel values



Idea: connect patches of input to neurons in hidden layer.
Neuron connected to region of input. Only "sees" these values.

Using Spatial Structure



Connect patch in input layer to a single neuron in subsequent layer.
Use a sliding window to define connections.
How can we **weight** *the patch to detect particular features?*

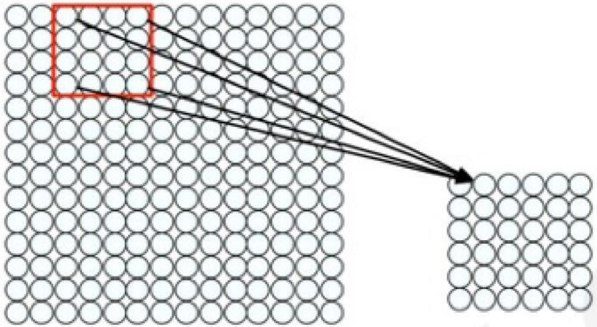
Applying Filters to Extract Features

1) Apply a set of weights – a filter – to extract **local features**

2) Use **multiple filters** to extract different features

3) Spatially **share** parameters of each filter
(features that matter in one part of the input should matter elsewhere)

Feature Extraction with Convolution



- Filter of size 4x4 : 16 different weights
- Apply this same filter to 4x4 patches in input
- Shift by 2 pixels for next patch

This "patchy" operation is **convolution**

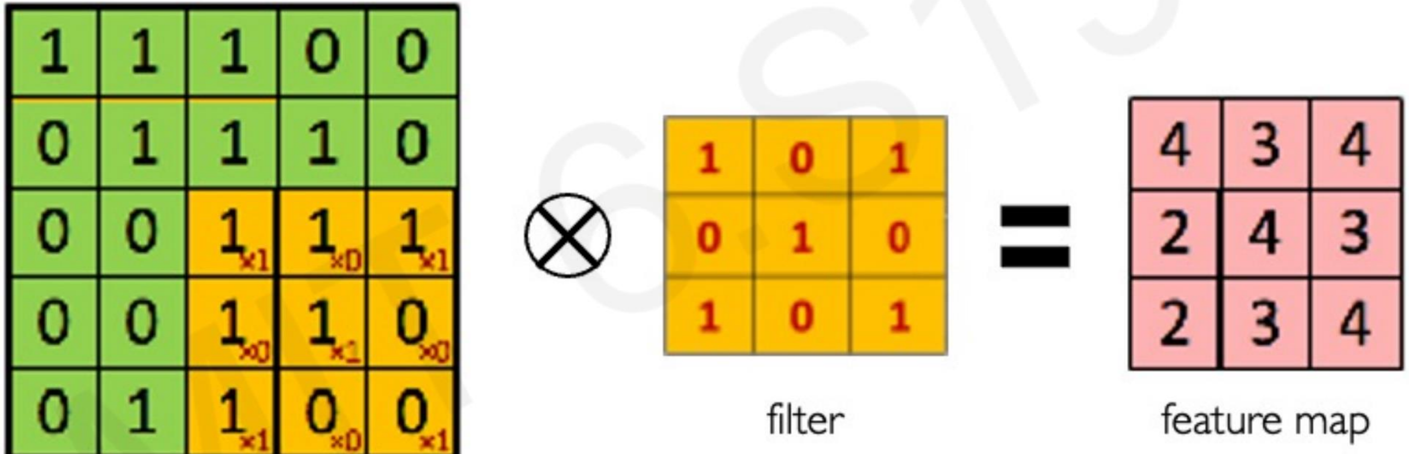
1) Apply a set of weights – a filter – to extract **local features**

2) Use **multiple filters** to extract different features

3) **Spatially share** parameters of each filter

The Convolution Operation

We slide the 3x3 filter over the input image, element-wise multiply, and add the outputs:

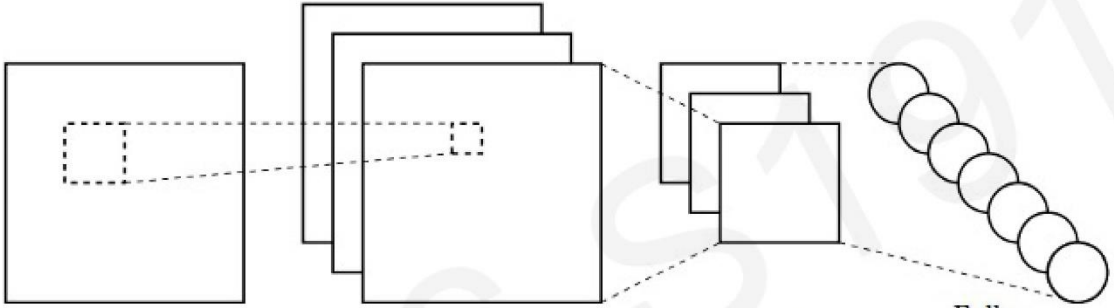


Producing Feature Maps

Original | Sharpen | Edge Detect | "Strong" Edge Detect

Convolutional Neural Networks (CNNs)

CNNs for Classification



1. **Convolution**: Apply filters to generate feature maps.
2. **Non-linearity**: Often ReLU.
3. **Pooling**: Downsampling operation on each feature map.

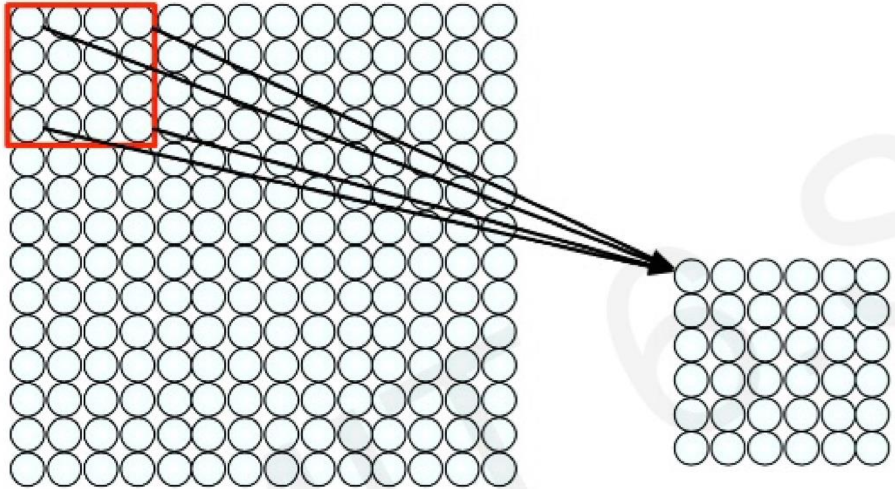
`tf.keras.layers.Conv2D`

`tf.keras.activations.*`

`tf.keras.layers.MaxPool2D`

Train model with image data.
Learn weights of filters in convolutional layers.

Convolutional Layers: Local Connectivity



`tf.keras.layers.Conv2D`

For a neuron in hidden layer:

- Take inputs from patch
- Compute weighted sum
- Apply bias

4x4 filter: matrix of weights w_{ij}

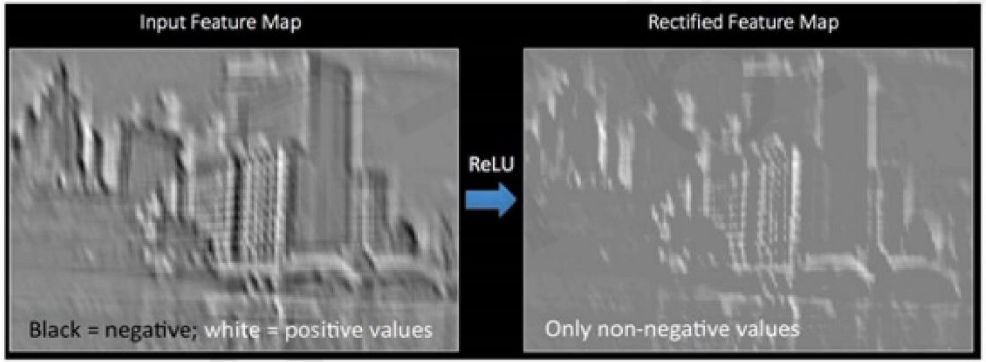
$$\sum_{i=1}^{4}\sum_{j=1}^{4} w_{ij}\, x_{i+p,j+q} + b$$

for neuron (p,q) in hidden layer

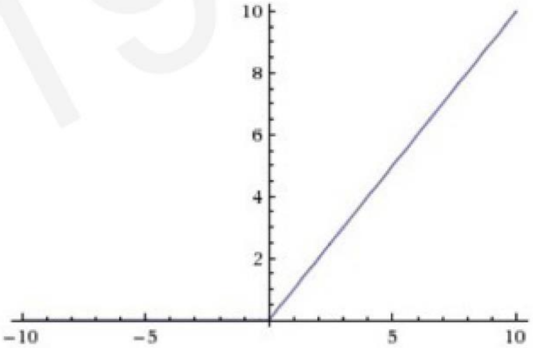
1) applying a window of weights
2) computing linear combinations
3) activating with non-linear function

Introducing Non-Linearity

- Apply after every convolution operation (i.e., after convolutional layers)
- ReLU: pixel-by-pixel operation that replaces all negative values by zero. **Non-linear operation!**



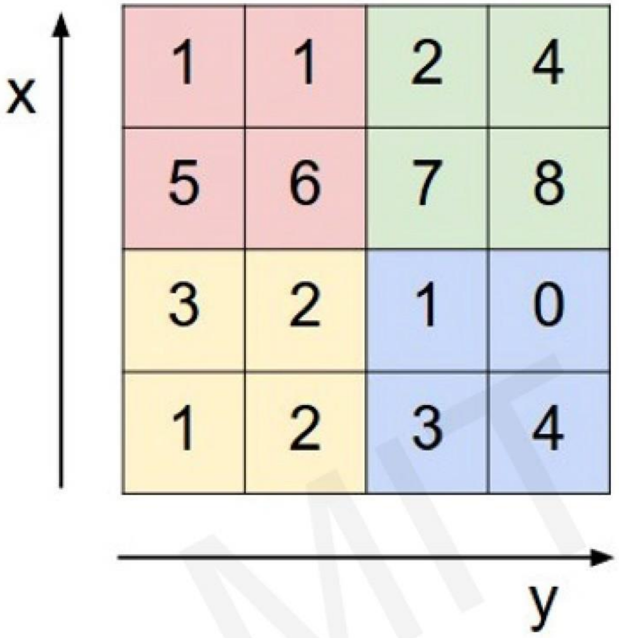
Rectified Linear Unit (ReLU)



$$g(z) = \max(0, z)$$

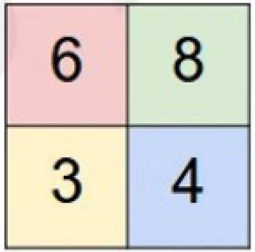
```
tf.keras.layers.ReLU
```

Pooling



max pool with 2x2 filters
and stride 2

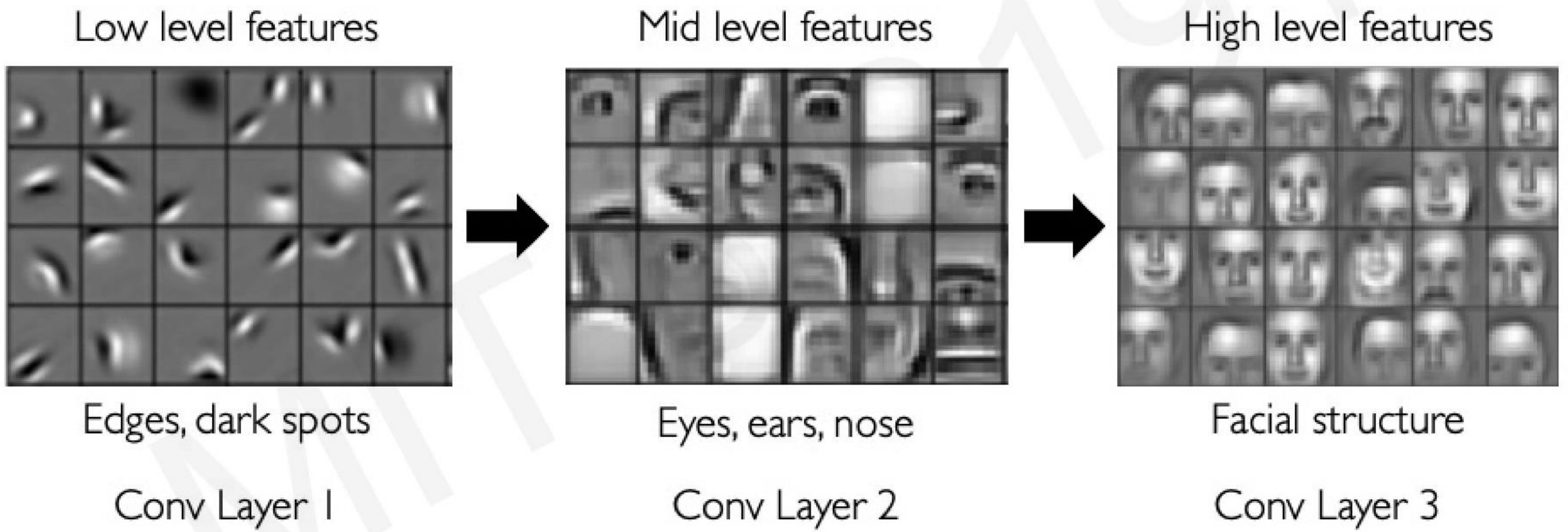
```
tf.keras.layers.MaxPool2D(
    pool_size=(2,2),
    strides=2
)
```



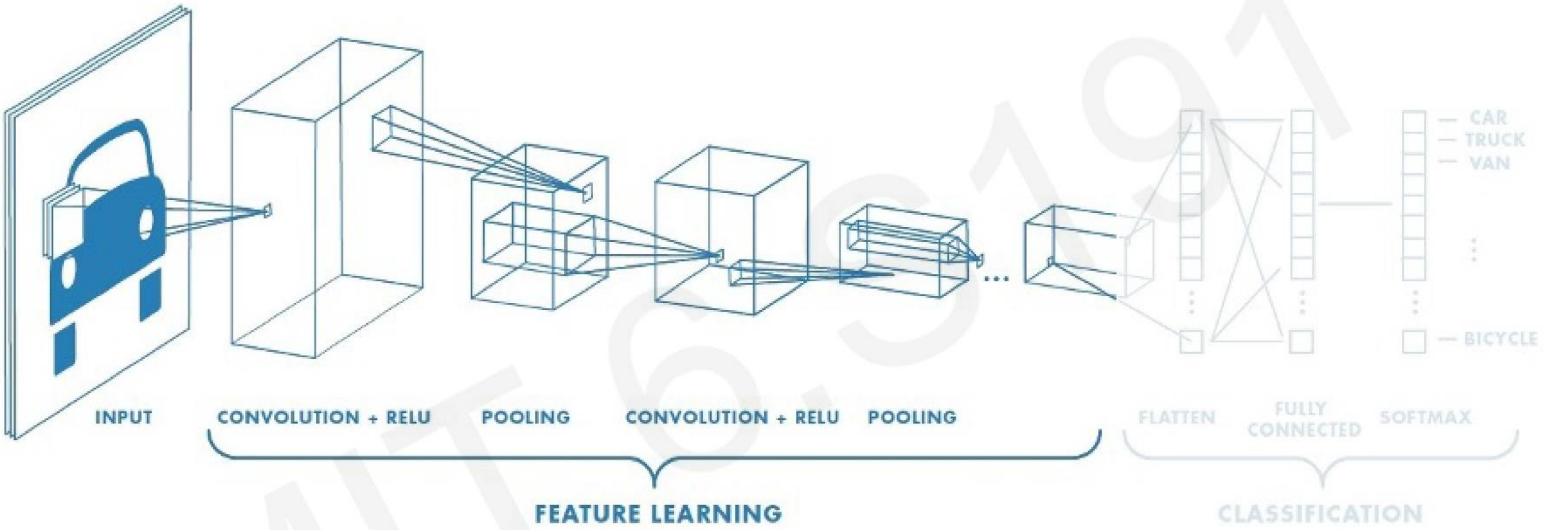
1) Reduced dimensionality
2) Spatial invariance

How else can we downsample and preserve spatial invariance?

Representation Learning in Deep CNNs

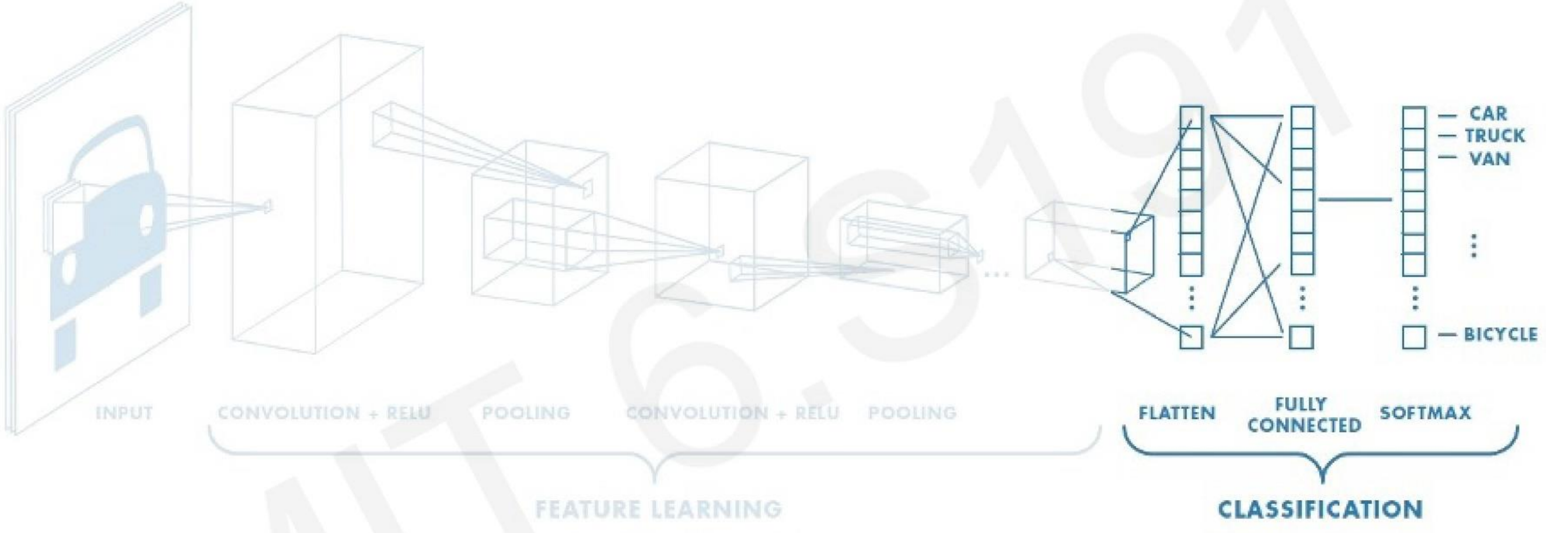


CNNs for Classification: Feature Learning



1. Learn features in input image through **convolution**
2. Introduce **non-linearity** through activation function (real-world data is non-linear!)
3. Reduce dimensionality and preserve spatial invariance with **pooling**

CNNs for Classification: Class Probabilities

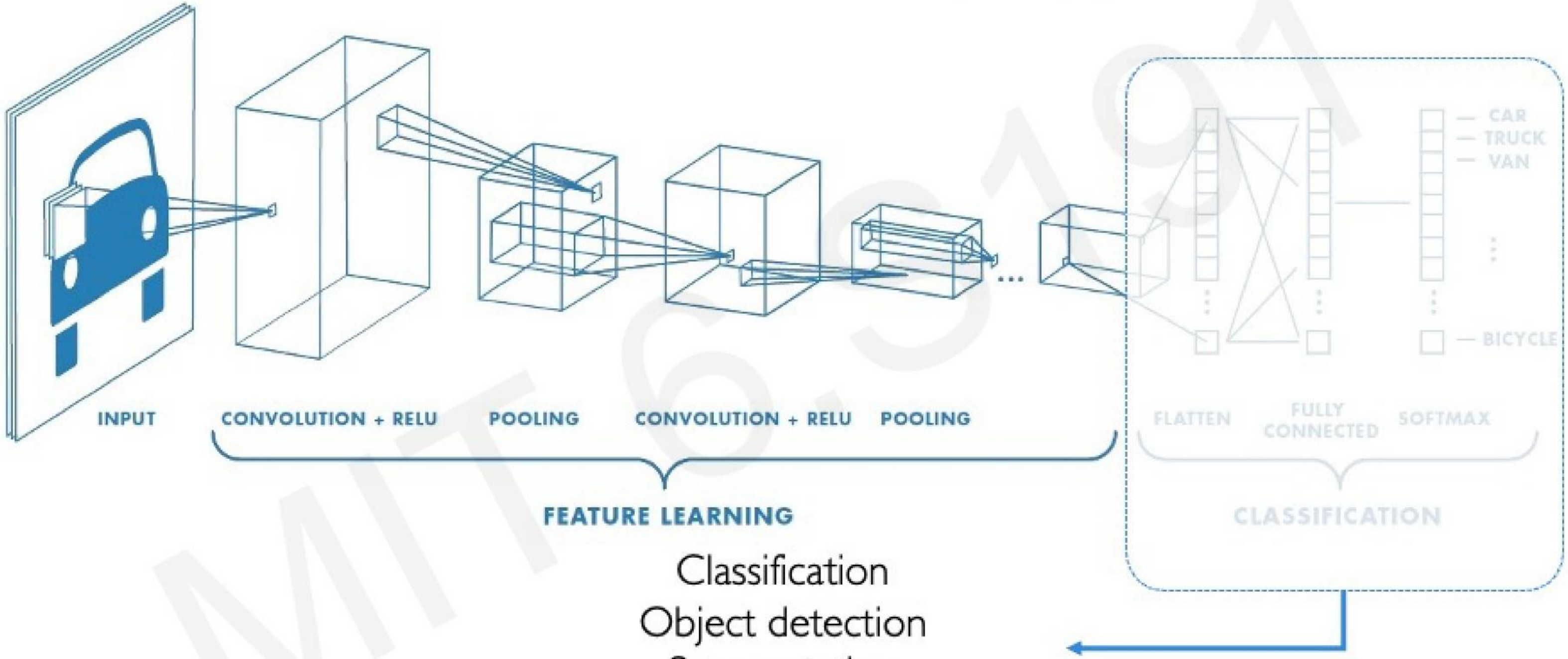


- CONV and POOL layers output high-level features of input
- Fully connected layer uses these features for classifying input image
- Express output as **probability** of image belonging to a particular class

$$\text{softmax}(y_i) = \frac{e^{y_i}}{\sum_j e^{y_j}}$$

人工智能基本理论

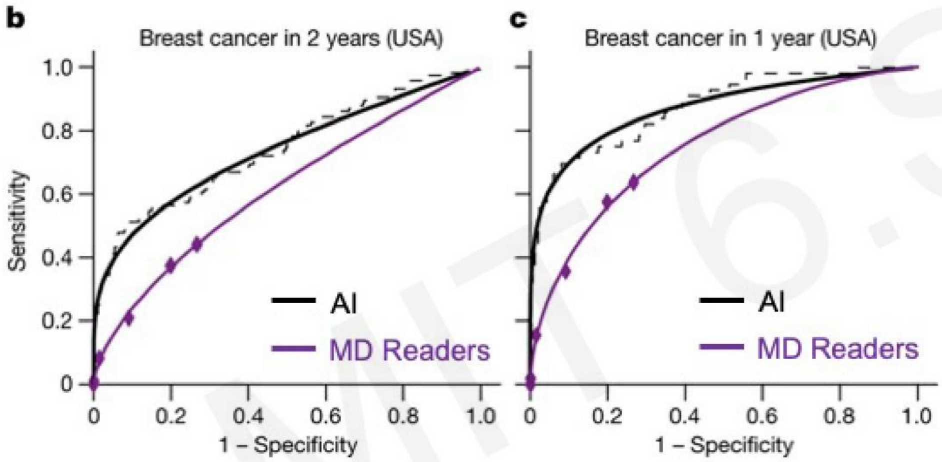
An Architecture for Many Applications



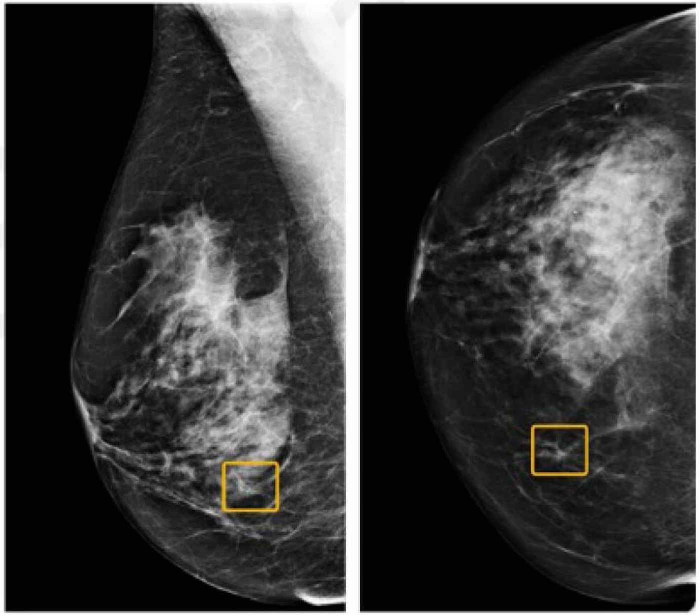
Classification
Object detection
Segmentation
Probabilistic control

Classification: Breast Cancer Screening

International evaluation of an AI system for breast cancer screening nature

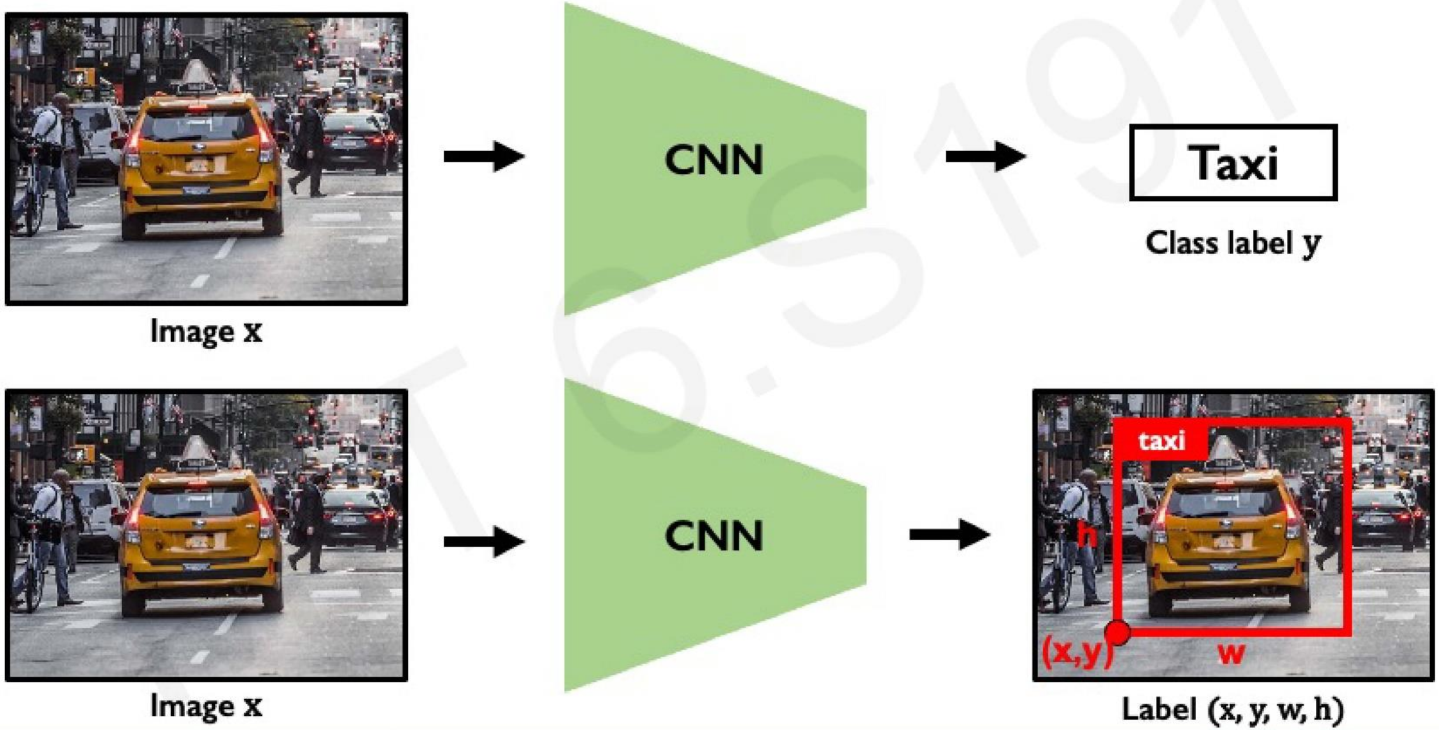


CNN-based system outperformed expert radiologists at detecting breast cancer from mammograms



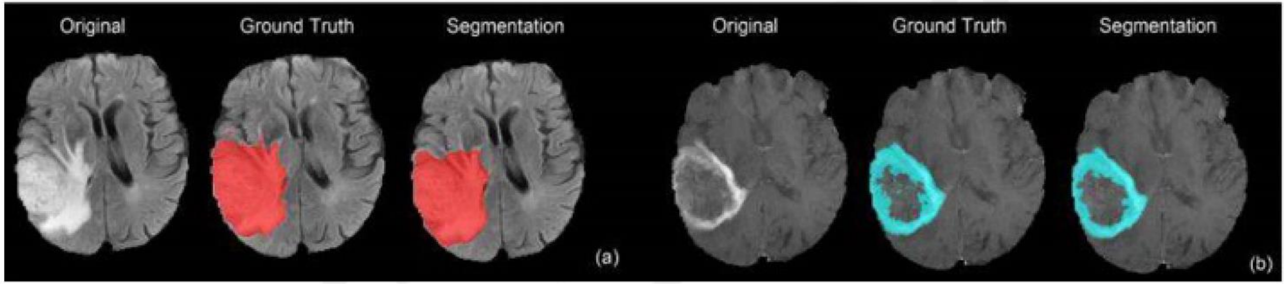
Breast cancer case missed by radiologist but detected by AI

Object Detection

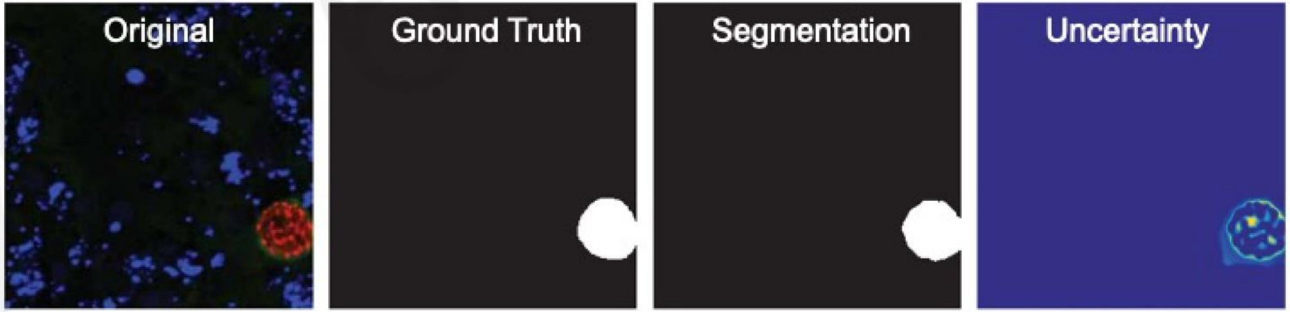


Semantic Segmentation: Biomedical Image Analysis

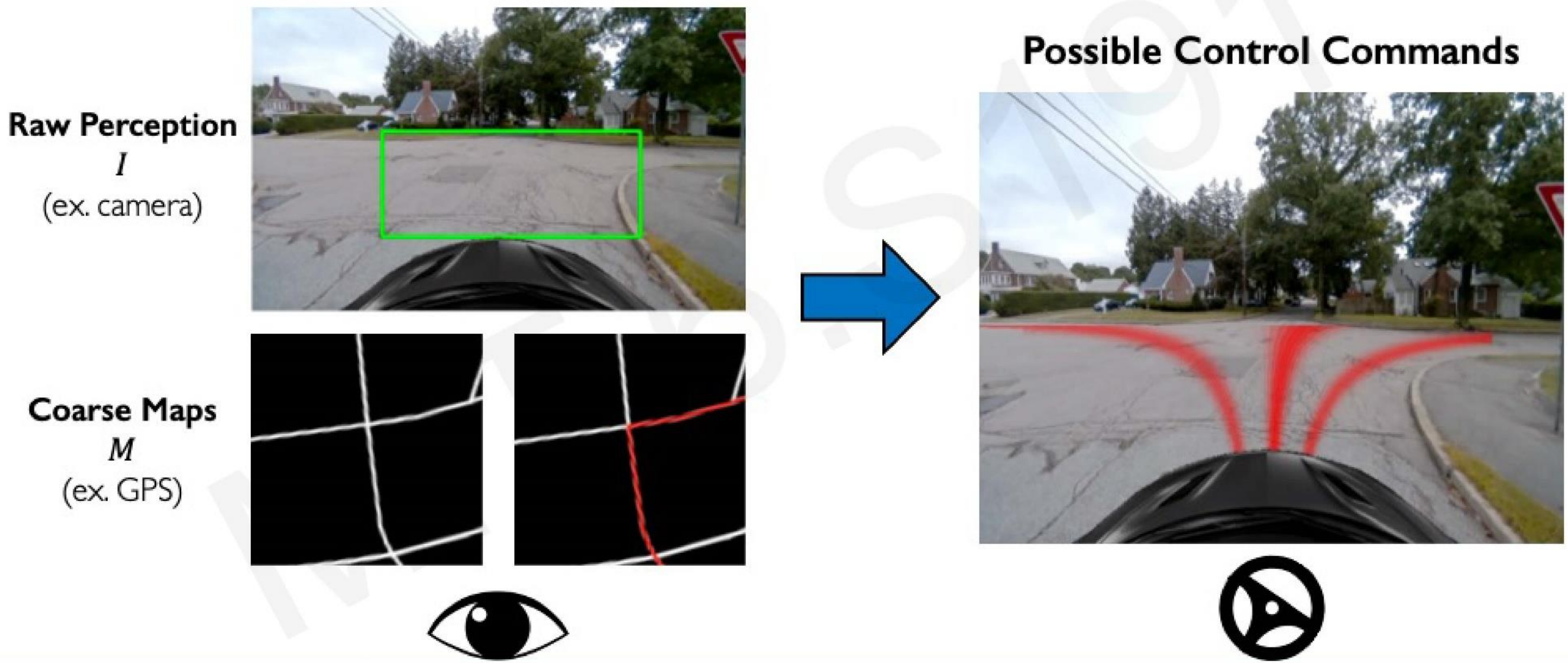
Brain Tumors
Dong+ *MIUA* 2017.



Malaria Infection
Soleimany+ *arXiv* 2019.

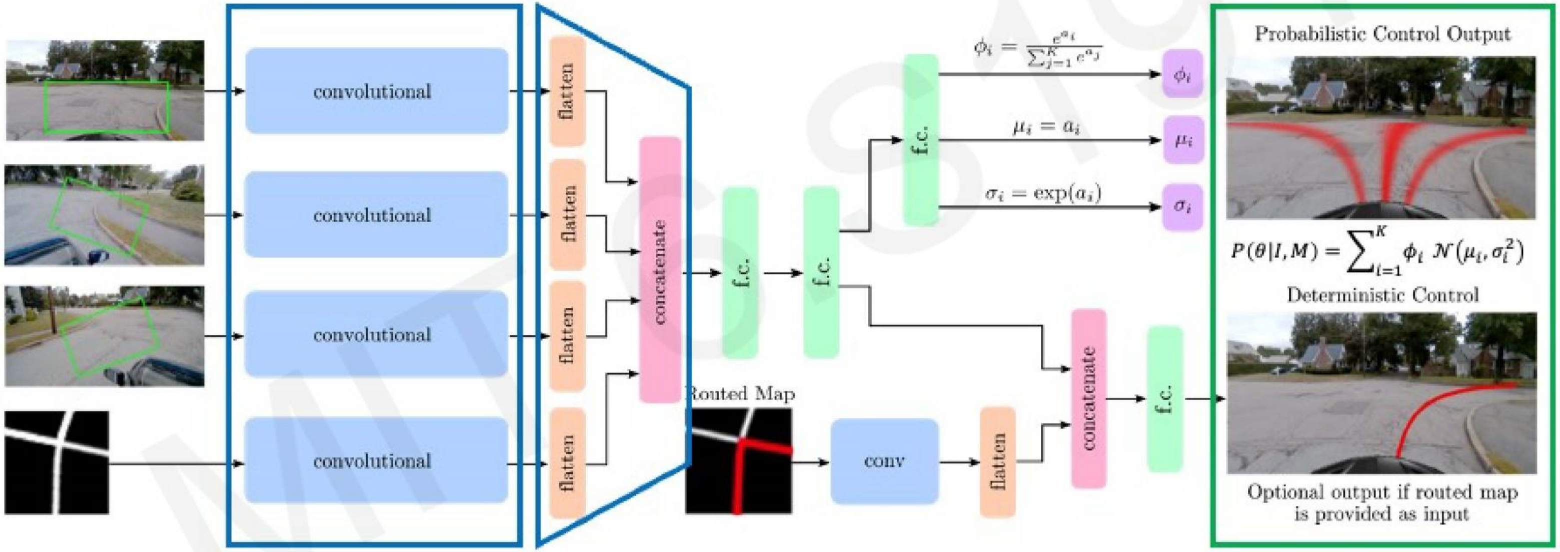


Continuous Control: Navigation from Vision



End-to-End Framework for Autonomous Navigation

Entire model is trained end-to-end **without any human labelling or annotations**



$$L = -\log(P(\theta|I,M))$$

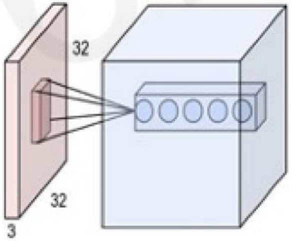
Deep Learning for Computer Vision: Summary

Foundations

- Why computer vision?
- Representing images
- Convolutions for feature extraction

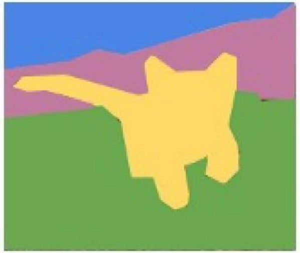
CNNs

- CNN architecture
- Application to classification
- ImageNet

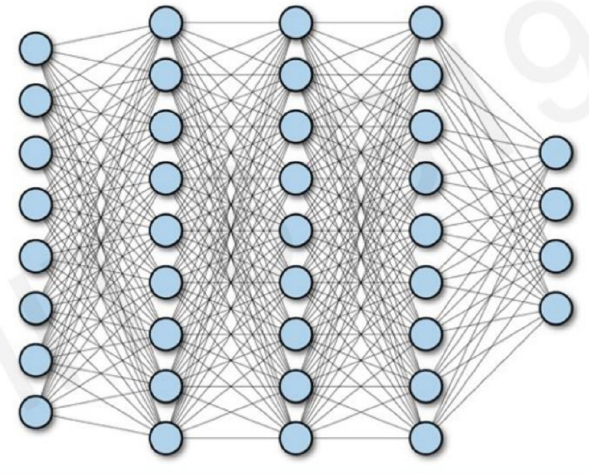


Applications

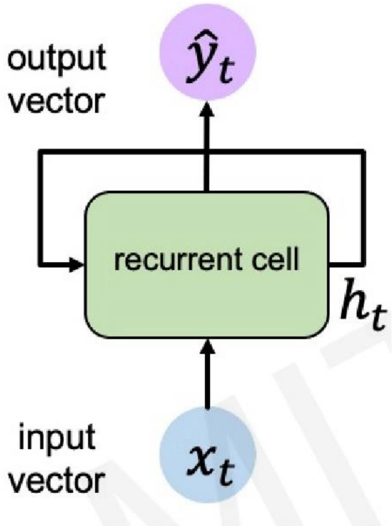
- Segmentation, image captioning, control
- Security, medicine, robotics



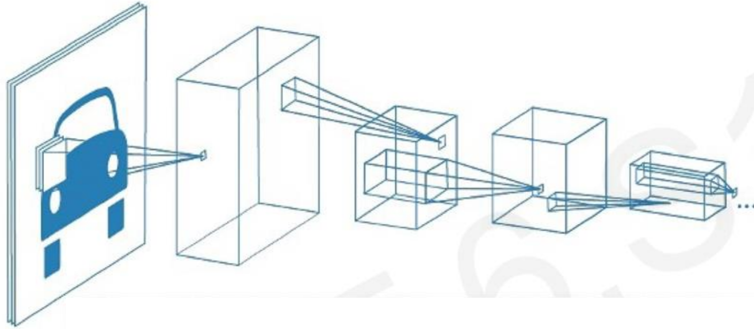
人工智能基本理论



前馈神经网络
MLP



循环神经网络
RNN



卷积神经网络
CNN

谢谢大家