Extended PID Controller for Nonminimum Phase Systems with Application to a Hypersonic Vehicle

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***Abstract*—In our previous work, we proposed the extended PID (EPID) controller, which is a state-space extension of traditional PID control. Compared to PID control, EPID is more suitable for multi-input-multi-output (MIMO) and higher-order systems. In this paper, we further extend EPID to nonminimum phase systems and investigate its performance limitation. EPID uses feedback of all state tracking errors. But for nonminimum phase systems, the reference trajectories for the internal states are unknown (assuming we do not have the system model), making us decide to take out the internal states from the integral part to avoid an unbounded input, which results in a slightly different controller form. Besides, in previous study, we found an important property of EPID is that it can achieve accurate tracking/rejecting for time-varying references/disturbances by using a high integral gain. However, when applied to nonminimum phase systems, we found that the integral gain cannot be set too high, otherwise the closed-loop system will be unstable, which indicates an inherent performance limitation. To verify this, simulation results are provided by applying EPID to a hypersonic vehicle model and a cart pole system.**

Keywords—PID Control, Nonminimum Phase Systems, Hypersonic Vehicle

I. Introduction

Proportional-integral-derivative (PID) control is perhaps the most popular controller in the control field. However, it does have some drawbacks in its current form such as the single-input-single-output (SISO) property, which makes it inconvenient to apply it to complex systems. To this end, we proposed the concept of extended PID (EPID) control in our previous work [1], which rewrites the PID controller in state space. Compared to PID control, EPID uses feedback of all state tracking errors instead of only using the output error, making it more suitable for multi-input-multi-output (MIMO) and higher-order systems. In [1], we reported an important property of EPID, that is, it can achieve accurate tracking/rejecting for time-varying references/disturbances by using a high integral gain. However, when it comes to nonminimum phase systems, this is no longer true.

In the control field, it has long been realized that the nonminimum phase property is a main restriction of control design. Nonminimum phase property not only makes it challenging to design a stable controller but also brings fundamental limitations to the control performance. Firstly, there is an inherent limitation in the tracking control error for nonminimum phase systems. It is shown in [2] that minimum phase is a necessary condition for asymptotic tracking for arbitrary trajectories. Reference [3] points out that it is possible to achieve perfect tracking when there are no unstable zero dynamics, that is, the L2 norm of the tracking error can be arbitrarily small. But it becomes impossible when there are unstable zero dynamics since some amount of "output energy" must be applied to stabilize the zero dynamics. Similarly, when tracking a step signal, the phenomenon of undershoot or overshoot [4] occurs for nonminimum phase systems, which depends on the unstable zeros. With the nonminimum phase zeros closer to the original and a shorter preview time of the future reference, the tracking performance is worse [5]. Besides, the nonminimum phase property also affects the bandwidth and robustness of the closed-loop system [6]. As we know, for linear systems, the poles of the closed-loop system move towards the open-loop zeros as the control gain increases. Therefore, for systems with unstable zeros, the control gain cannot be too large to maintain stability. This restricts the application of high-gain feedback to nonminimum phase systems and thus limits the robustness of the system.

The application of PID control to nonminimum phase systems has also been studied, where many of them focused on parameter tuning. Due to the instability of nonminimum phase systems, it is important to select proper PID parameters to ensure stability. In [7], a PID controller tuned by the gain-phase margin method is proposed for a non-minimum phase system with an uncertain time delay. In [8], a tuning method is developed for the stabilization of non-minimum phase second-order plus time delay systems. In [9], a PID controller is designed for a class of fourth-order nonminimum-phase systems and a parameter tuning method is proposed based on Routh-Hurwitz criteria. In [10], a PID controller is designed for a non-minimum phase DC-DC converter, and the parameters are tuned to achieve better robustness by using the quantitative feedback theory along with particle swarm optimization. In [11], a method is proposed to tune the PID controller for MIMO systems. However, all the work above are still based on the traditional PID control framework which uses transfer function as the main tool for analysis. While in the EPID framework, full-state feedback is used and thus stability can be easily guaranteed by using pole assignment to determine the control gain matrix. However, the influence of the integral gain still needs to be studied.

In this paper, we extends EPID to nonminimum phase systems and then investigate its performance limitations using examples of two nonminimum phase systems, including a hypersonic vehicle model and a cart pole system. Results show some interesting properties. On one hand, the integral gain cannot be set too high for nonminimum phase systems, otherwise the closed-loop system will be unstable. On the other hand, although the performance limitation exists, using integral control is still very useful to reduce steady-state error for some weak nonminimum phase systems such as the hypersonic vehicle. This paper enriches the EPID framework and the obtained results may be helpful when designing EPID controller for other nonminimum phase systems.

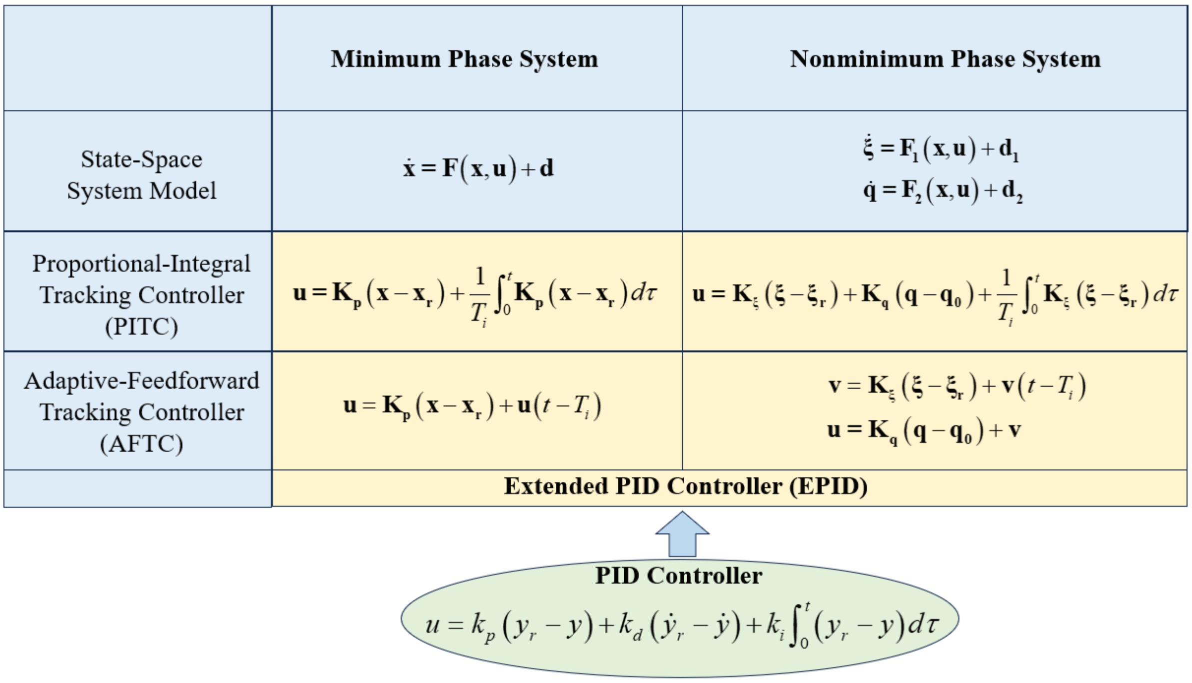

Fig. 1 The EPID control framework.

II. The EPID Control Framework

The EPID control framework proposed in this paper is shown in Fig. 1. EPID has two equivalent forms, including a proportional-integral tracking controller (PITC) and an adaptive-feedforward tracking controller (AFTC). EPID has slightly different forms when applied to minimum-phase and nonminimum-phase systems.

A. *EPID for Minimum Phase Systems*

The state-space model of a minimum phase system can be generally written as follows:

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x},\mathbf{u}) + \mathbf{d} \tag{1}$$

where $\mathbf{u} = [u_1, u_2, ..., u_m]^T$ is the input, $\mathbf{y} = [y_1, y_2, ..., y_m]^T$ is the output, $\mathbf{x} = \left[y_1, \dot{y}_1, ..., y_1^{(r_1-1)}, ..., y_m, \dot{y}_m, ..., y_m^{(r_m-1)}\right]^T$ is the state ($r_1, r_2, ..., r_m$ represent the relative degree of each output), $\mathbf{d} = [d_1, d_2, ..., d_m]^T$ is the external disturbance.

For this system, when the output reference $\mathbf{y_r}(t) = [y_{1r}(t), y_{2r}(t), ..., y_{mr}(t)]^T$ are given, then all the state references are known, that is, $\mathbf{x_r} = \left[y_{1r}, \dot{y}_{1r}, ..., y_{1r}^{(r_1-1)}, ..., y_{mr}, \dot{y}_{mr}, ..., y_{mr}^{(r_m-1)}\right]^T$.

By using the state error instead of the output error in PID, the EPID controller is written as follows

$$\mathbf{u} = \mathbf{K_p}(\mathbf{x}-\mathbf{x_r}) + \frac{1}{T_i}\int_0^t \mathbf{K_p}(\mathbf{x}-\mathbf{x_r})d\tau \tag{2}$$

where $\mathbf{K_p}$ is the proportional gain matrix and T_i is the integral time. We call this form PITC. The derivative part is merged into the proportional part since the state error $\mathbf{x}-\mathbf{x_r}$ includes the output derivative when the relative degree > 1.

EPID can be also written in another form as follows

$$\mathbf{u} = \mathbf{K_p}(\mathbf{x}-\mathbf{x_r}) + \mathbf{u}(t-T_i) \tag{3}$$

where the input is a sum of the previous input with a time delay and the state error feedback. We call this form AFTC.

B. *EPID for Nonminimum Phase Systems*

A general form of a MIMO nonminimum phase system can be written as follows:

$$\begin{aligned} \dot{\boldsymbol{\xi}} &= \mathbf{F}_1(\mathbf{x},\mathbf{u}) + \mathbf{d}_1 \\ \dot{\mathbf{q}} &= \mathbf{F}_2(\mathbf{x},\mathbf{u}) + \mathbf{d}_2 \end{aligned} \tag{4}$$

where $\mathbf{x} = [\boldsymbol{\xi}, \mathbf{q}]^T$ is the state with $\mathbf{q}$ being the internal state and $\boldsymbol{\xi} = \left[y_1, \dot{y}_1, ..., y_1^{(r_1-1)}, ..., y_m, \dot{y}_m, ..., y_m^{(r_m-1)}\right]^T$ being the external state.

In this system, the internal dynamics are not stable. Therefore $\mathbf{q}$ must be used in the feedback control to ensure stability. However, the problem is that the internal state reference $\mathbf{q_r}$ is not known. In fact, $\mathbf{q_r}$ should be a bounded solution of the unstable zero dynamics, which is called *ideal internal dynamics* (IID) [12]. If the system model is known, then $\mathbf{q_r}$ can be solved by using stable inversion [13], output regulation [14], or optimal bounded inversion [15]. However, in the EPID framework, we emphasize using minimum system information. Therefore, the exact solution of $\mathbf{q_r}$ will not be part of the EPID framework. We are interested in how to extend EPID to nonminimum phase systems without calculating $\mathbf{q_r}$. Anyhow, the unknown of exact $\mathbf{q_r}$ is the first factor that prevents the nonminimum phase system from accurate tracking.

Our method is to use a constant $\mathbf{q_0}$ as the best guess for $\mathbf{q_r}$. The value $\mathbf{q_0}$ can be decided by our experience with the operation range of the variable $\mathbf{q}$, such as choosing the median or directly setting to $\mathbf{0}$ if the operation range is small.

The second step is to determine the controller structure. We may imagine using the same EPID structure designed for minimum phase system by taking $\mathbf{x_r}=[\boldsymbol{\xi}_\mathbf{r},\mathbf{q_0}]^T$. But this is not feasible. Recall the AFTC $\mathbf{u}=\mathbf{K_p}(\mathbf{x}-\mathbf{x_r})+\mathbf{u}(t-T_i)$, consider constant reference/disturbance, then we have $\mathbf{K_p}(\mathbf{x}-\mathbf{x_r})=\mathbf{K_p}\begin{bmatrix}\boldsymbol{\xi}-\boldsymbol{\xi}_\mathbf{r}\\ \boldsymbol{\eta}-\boldsymbol{\eta}_\mathbf{0}\end{bmatrix}=\mathbf{0}$ at steady state. However, this does not indicate $\mathbf{x}-\mathbf{x_r}=\mathbf{0}$ since $\mathbf{q_0}$ possibly does not equal the IID which means $\boldsymbol{\eta}\neq\boldsymbol{\eta}_\mathbf{0}$ and thus $\boldsymbol{\xi}-\boldsymbol{\xi}_\mathbf{r}\neq\mathbf{0}$. Therefore, although an integral term is used, there may still exist a steady state error. To solve this problem, we use the following PITC

$$\mathbf{u}=\mathbf{K}_\xi(\boldsymbol{\xi}-\boldsymbol{\xi}_\mathbf{r})+\mathbf{K_q}(\mathbf{q}-\mathbf{q_0})+\frac{1}{T_i}\int_0^t\mathbf{K}_\xi(\boldsymbol{\xi}-\boldsymbol{\xi}_\mathbf{r})d\tau \tag{5}$$

and the corresponding AFTC

$$\begin{aligned}\mathbf{v}&=\mathbf{K}_\xi(\boldsymbol{\xi}-\boldsymbol{\xi}_\mathbf{r})+\mathbf{v}(t-T_i)\\ \mathbf{u}&=\mathbf{K_q}(\mathbf{q}-\mathbf{q_0})+\mathbf{v}\end{aligned} \tag{6}$$

For this controller, we have $\mathbf{v}=\mathbf{v}(t-T_i)$ at steady state and thus $\mathbf{K}_\xi(\boldsymbol{\xi}-\boldsymbol{\xi}_\mathbf{r})=\mathbf{0}$, so $\boldsymbol{\xi}-\boldsymbol{\xi}_\mathbf{r}=0$ and there will be no steady-state error. It is compatible with the previous EPID if we remove the internal state feedback $\mathbf{K_q}(\mathbf{q}-\mathbf{q_0})$.

Therefore, the EPID framework is completed for both minimum and nonminimum phase systems. However, it is well known that nonminimum phase systems have fundamental performance limitations. Reflecting in EPID is that the integral gain cannot be set too high, otherwise the closed-loop system will be unstable. This is the second factor that prevents the nonminimum phase system from good tracking. We will show this in the simulations.

III. The Hypersonic Vehicle Model

A hypersonic vehicle (Fig. 2) refers to a vehicle that travels at least five times the speed of sound. In this paper, we consider a flexible hypersonic vehicle model [16].

Fig. 2 A typical hypersonic vehicle: the X-43.

According to [16], the longitudinal dynamics of the vehicle are written as follows

$$\begin{cases}\dot{V}=(T\cos\alpha-D-mg\sin\gamma)/m\\ \dot{h}=V\sin\gamma\\ \dot{\gamma}=(L+T\sin\alpha-mg\cos\gamma)/(mV)\\ \dot{\theta}=Q\\ \dot{Q}=M/I_{yy}\\ \ddot{\eta}_i=-2\zeta_i\omega_i\dot{\eta}_i-\omega_i^2\eta_i+N_i,\ i=1,2,3\end{cases} \tag{7}$$

The model is composed of five rigid-body states $\mathbf{x}=[V,h,\gamma,\theta,Q]^T$, representing velocity, altitude, flight path angle, pitch angle, and pitch rate, respectively, and six flexible states $\boldsymbol{\eta}=[\eta_1,\dot{\eta}_1,\eta_2,\dot{\eta}_2,\eta_3,\dot{\eta}_3]^T$ corresponding to the first three flexible modes of the fuselage. There are three control inputs $\mathbf{u}=[\phi,\delta_e,\delta_c]^T$, representing fuel to air ratio in the scramjet engine, the elevator deflection angle, and the canard deflection angle, respectively. The control inputs indirectly affect the dynamics through the forces and moments T, D, L, M and N_i with the expressions given by

$$\begin{aligned}&T=\bar{q}S[C_T(\alpha)+C_{T\phi}(\alpha)\phi+\mathbf{C_T^\eta}\boldsymbol{\eta}]\\ &L=\bar{q}S[C_L(\alpha)+C_L^{\delta_e}\delta_e+\mathbf{C_L^\eta}\boldsymbol{\eta}]\\ &D=\bar{q}S[C_D(\alpha)+C_D^{\delta_e^2}\delta_e^2+C_D^{\delta_e}\delta_e+C_D^{\delta_c^2}\delta_c^2+C_D^{\delta_c}\delta_c+\mathbf{C_D^\eta}\boldsymbol{\eta}]\\ &M=z_TT+\bar{q}\bar{c}S[C_M(\alpha)+C_M^{\delta_e}\delta_e+C_M^{\delta_c}\delta_c+\mathbf{C_M^\eta}\boldsymbol{\eta}]\\ &N_i=\bar{q}S[N_i^{\alpha^2}\alpha^2+N_i^{\alpha}\alpha+N_i^{\delta_e}\delta_e+N_i^{\delta_c}\delta_c+N_i^0+\mathbf{N_i^\eta}\boldsymbol{\eta}],\ i=1,2,3\end{aligned} \tag{8}$$

where

$$\begin{aligned}&C_T(\alpha)=C_T^3\alpha^3+C_T^2\alpha^2+C_T^1\alpha+C_T^0\\ &C_{T\phi}(\alpha)=C_T^{\phi\alpha^3}\alpha^3+C_T^{\phi\alpha^2}\alpha^2+C_T^{\phi\alpha}\alpha+C_T^{\phi}\\ &C_L(\alpha)=C_L^{\alpha}\alpha+C_L^0\\ &C_D(\alpha)=C_D^{\alpha^2}\alpha^2+C_D^{\alpha}\alpha+C_D^0\\ &C_M(\alpha)=C_M^{\alpha^2}\alpha^2+C_M^{\alpha}\alpha+C_M^0\\ &\mathbf{C_j^\eta}=\begin{bmatrix}C_j^{\eta_1} & 0 & C_j^{\eta_2} & 0 & C_j^{\eta_3} & 0\end{bmatrix},\ j=T,M,L,D\\ &\mathbf{N_i^\eta}=\begin{bmatrix}N_j^{\eta_1} & 0 & N_j^{\eta_2} & 0 & N_j^{\eta_3} & 0\end{bmatrix},i=1,2,3\end{aligned} \tag{9}$$

The angle of attack α satisfies $\alpha=\theta-\gamma$. The dynamic pressure $\bar{q}$ is calculated by $\bar{q}=\rho(h)V^2/2$ with $\rho(h)=\rho_0\,\mathrm{e}^{-(h-h_0)/h_s}$ being the atmospheric density. The values for the model parameters can be found in [16].

For the hypersonic vehicle model (7), it is fully actuated and minimum phase when the canard is enabled (i.e., δ_c available). However, when the canard is disabled, (i.e., $\delta_c=0$), it becomes a nonminimum phase system [16]. This provides us a good chance to test EPID in both cases and compare the performances to explore the influence of the nonminimum phase property.

IV. Simulation Results

A. Hypersonic Vehicle with Canard

When the canard is on, the outputs are $\mathbf{y}=[V,h,\theta]^T$ and the inputs are $\mathbf{u}=[\phi,\delta_e,\delta_c]^T$, the references are $\mathbf{y_r}=[V_r,h_r,\theta_r]^T$. The flexible states are treated as disturbance and the state $\mathbf{x}=[V,h,\gamma,\theta,Q]^T$ is used in the EPID controller.

The initial condition is $\mathbf{x}(0)=[7850,86000,0,0.0237,0]^T$, $\boldsymbol{\eta}(0)=\mathbf{0}$. The altitude reference h_r is generated by filtering a step command (from 86000 ft to 99000 ft) with two second-order filters with a natural frequency of $\omega_f = 0.03 rad/s$ and a damping factor of $\zeta_f = 0.95$. The velocity reference is computed by $V_r = \left[2\bar{q}\exp\left((h_r - h_0)/h_s\right)/\rho_0\right]^{1/2}$ to maintain a constant dynamic pressure. The flight path angle reference is obtained by $\gamma_r = \operatorname{asin}\left(\dot{h}_r / V_r\right)$. The pitch angle reference is given by $\theta_r = 0.0237 + \gamma_r$ and pitch rate velocity $Q_r = \dot{\theta}_r$. So all the state references $\mathbf{x_r} = [V_r, h_r, \gamma_r, \theta_r, Q_r]^T$ are known.

The control inputs are limited by $\phi \in [0, 1.5]$, $\delta_e, \delta_c \in [-20, 20]$deg. The control parameters are selected as $\mathbf{K_P} = \begin{bmatrix} -0.312 & 0 & 1.67 & -1.84 & -0.146 \\ -0.000989 & -0.0521 & -125 & 0.357 & 0.256 \\ 0.00531 & -0.026 & -60.6 & -8.38 & -1.39 \end{bmatrix}$. Two integral gains are tested, $k_i = 0$ and $k_i = 100$.

The simulation results for the three outputs are shown in Figs. 3-5. It can be seen that a steady-state error exists for each output without the integral control. With a large integral gain $k_i = 100$, the system still keeps stable and all the tracking errors converge to zero rapidly with a minor initial error. The performance is comparable with the nonlinear controller designed in [16].

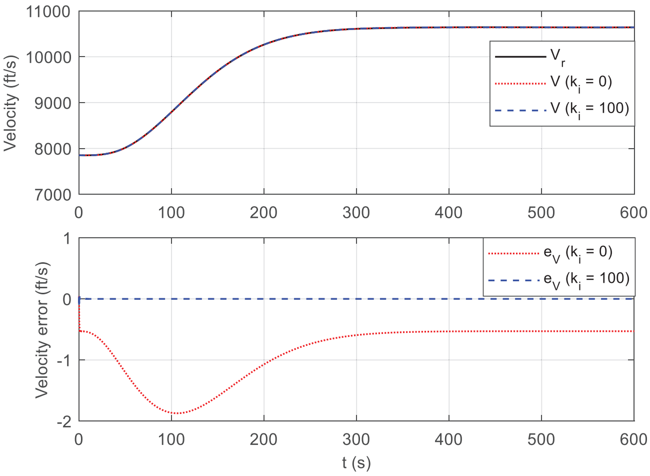

Fig. 3 Velocity tracking results.

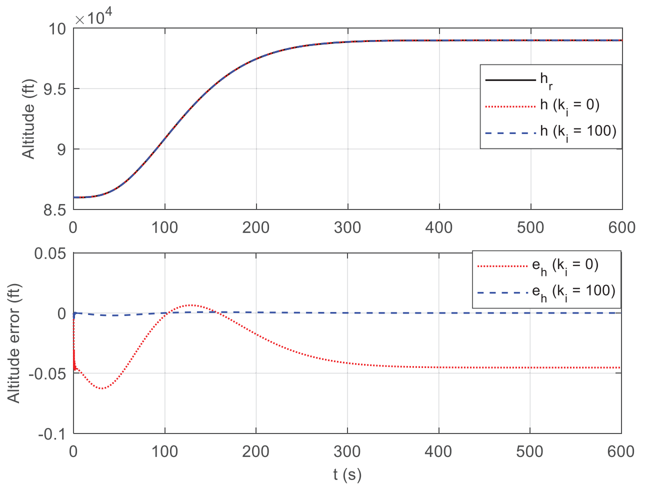

Fig. 4 Altitude tracking results.

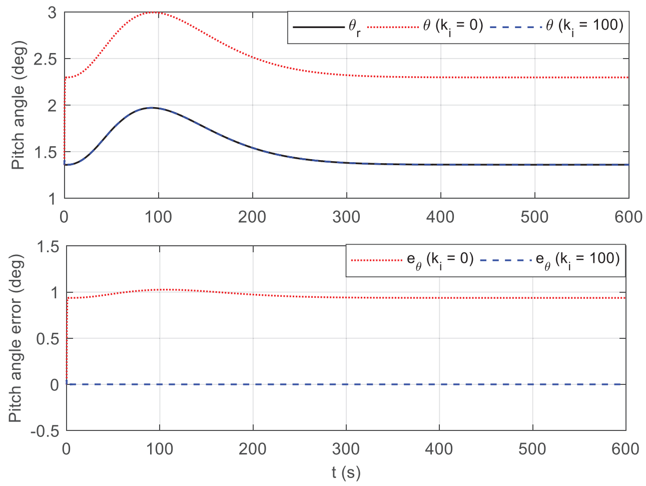

Fig. 5 Pitch angle tracking results.

The control inputs are shown in Fig. 6. It can be seen that the inputs have some oscillation at the beginning, then change smoothly throughout the task.

The results are consistent with our conclusion in [1] that a high integral gain can be set in EPID to achieve accurate tracking for minimum phase systems.

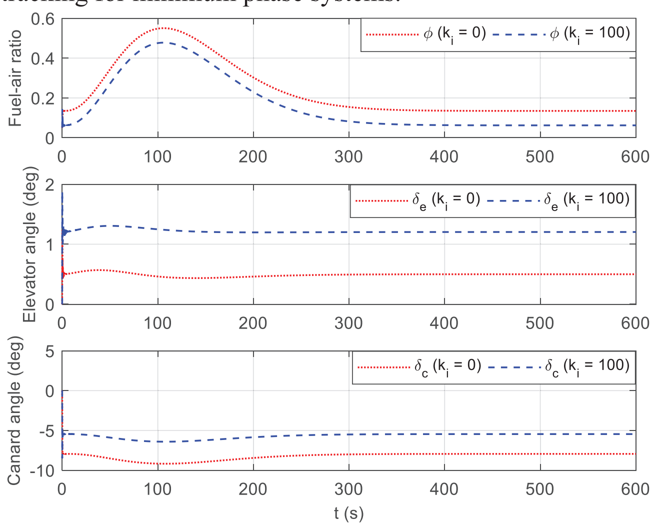

Fig. 6 Control inputs.

B. Hypersonic Vehicle without Canard

Now assume that the vehicle has no canard, that is, δ_c is set to zero in the model. Then it becomes a nonminimum phase system. The outputs are $\mathbf{y} = [V, h]^T$ now. The external states are $\boldsymbol{\xi} = [V, h, \gamma]^T$ and internal states are $\mathbf{q} = [\theta, Q]^T$. The control parameters are designed as $\mathbf{q_0} = \mathbf{0}$, $\mathbf{K_\xi} = \begin{bmatrix} -0.036, -0.042, -320 \\ -0.0037, -0.002, -19.7 \end{bmatrix}$, and $\mathbf{K_q} = \begin{bmatrix} -51, -8.7 \\ 3.1, \ 0.78 \end{bmatrix}$. The simulation results are shown in Figs. 7-8.

It can be observed that there is a big steady-state error for both outputs when $k_i = 0$. As k_i increases to 1, the steady-state error is eliminated, and the overall tracking performance has a significant improvement. However, we also test $k_i = 2$ and bigger values but find them unstable.

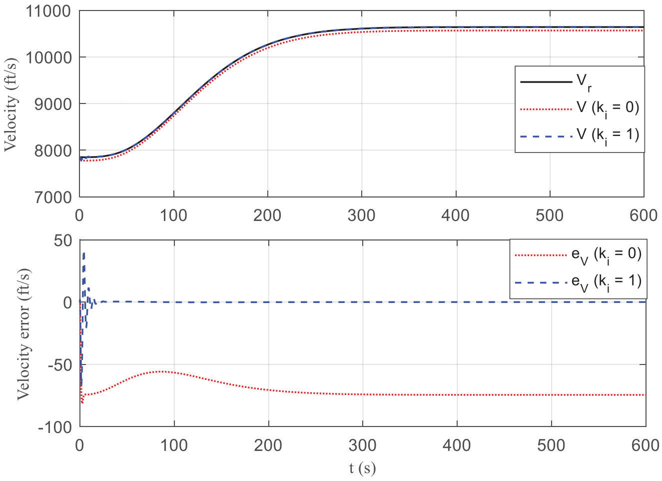

Fig. 7 Velocity tracking results (no canard).

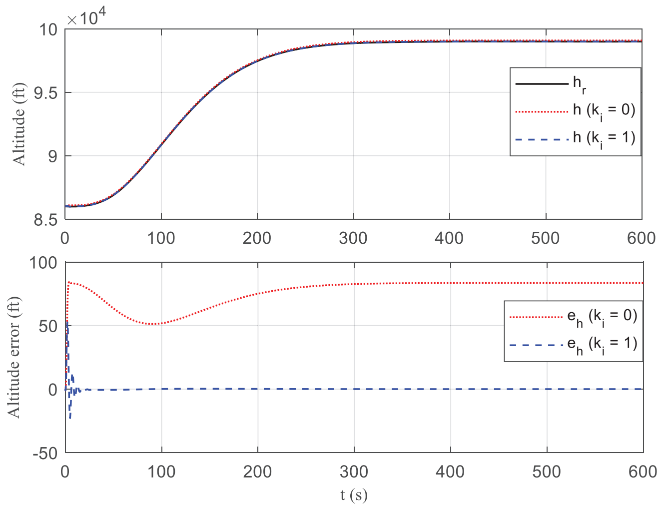

Fig. 8 Altitude tracking results (no canard).

V. Application to a Cart-Pole System

To further test EPID, we consider another nonminimum phase system, a cart-pole system as shown in Fig. 9.

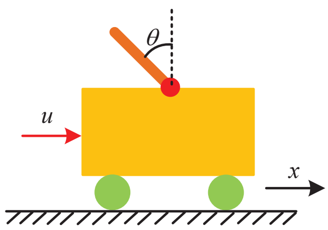

Fig. 9 The cart-pole system.

The model is

$$
\begin{aligned}
\dot{x} &= v \\
\dot{v} &= \frac{3mg\sin\theta\cos\theta + 4ml\omega^2\sin\theta + 4u}{4(M+m) - 3m\cos^2\theta} \\
\dot{\theta} &= \omega \\
\dot{\omega} &= \frac{-3(M+m)g\sin\theta - 3ml\omega^2\sin\theta\cos\theta - 3\cos\theta u}{4(M+m)l - 3ml\cos^2\theta}
\end{aligned}
\tag{10}
$$

where u is an external force imposed on the cart, x is the cart position, θ is the pole angle, m and M are the mass of the pole and the cart, respectively, l is the half length of the pole.

For this system, the external and internal states are $\boldsymbol{\xi} = [x, v]^T$ and $\mathbf{q} = [\theta, \omega]^T$, respectively. The output is x.

In the simulation, the model parameters are selected as $m = 1kg$, $M = 10kg$, $l = 1m$, and $g = 9.8m/s^2$. The initial condition is set to zero for all states. As a practical consideration, the control input is assumed to be limited in the range $u \in [-30, 30]$. The control parameters are selected as $\mathbf{q_0} = [0,0]^T$, $\mathbf{K}_\xi = [-5, -10]$, and $\mathbf{K_q} = [250, 80]$.

The simulation results are shown in Figs. 10-11. From Fig. 10, it can be seen that the pole is stabilized around the vertical up position ($\theta = 0$) for $k_i = 0$ and $k_i = 0.2$, but exhibits a divergence when $k_i = 0.7$. From Fig. 11, it can be observed that the tracking performance is not good. The system becomes unstable when $k_i = 0.7$. And a smaller integral gain $k_i = 0.2$ does not help improve the tracking performance but brings more oscillation compared to $k_i = 0$. This further verifies the performance limitations for nonminimum phase systems.

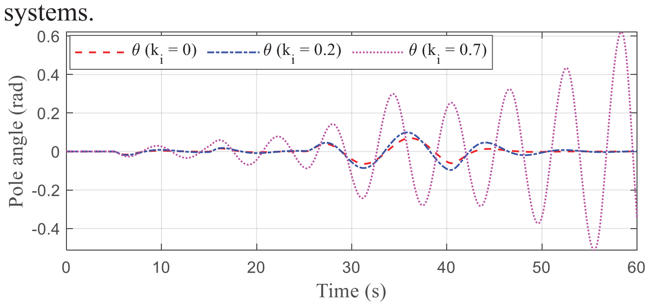

Fig. 10 Pole angle.

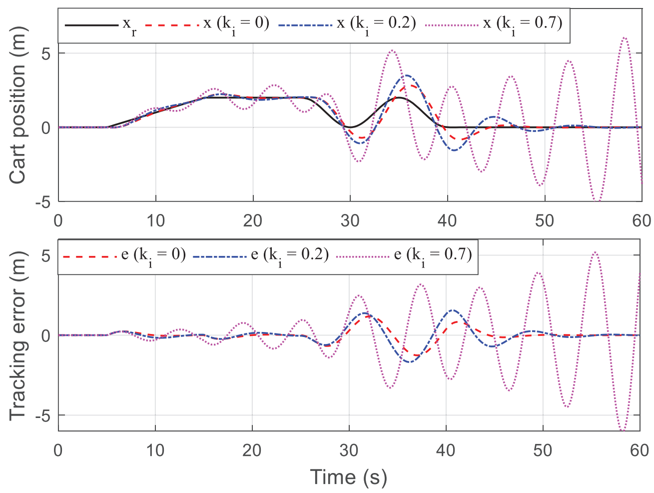

Fig. 11 Cart position and tracking error.

VI. Discussion

Comparing the hypersonic vehicle and cart pole examples, they both show that the integral gain cannot be too high for nonminimum phase systems. So we would like to ask if a small integral gain is helpful to improve the tracking performance. The answer is not sure as pointed out by the two examples, where a small integral gain helps in the hypersonic vehicle example but does not in the cart-pole example. There are two possible explanations. On one hand, the reference trajectory for the hypersonic vehicle changes much more slowly than that for the cart-pole system. The tracking performance for the hypersonic vehicle may get worse if a more aggressive reference is selected. So performance limitations still exist. On the other hand, a cart-pole system is probably a strongly nonminimum phase system [17] while the hypersonic vehicle is a slightly nonminimum phase system [18]. Therefore, the performance limitations are more severe for the cart-pole system. In practice, we should decide whether to use the integral action for a nonminimum phase system based on the actual situation.

VII. Conclusions

In this paper, we complete the EPID control framework by proposing its variation for nonminimum phase systems. Compared to PID, EPID is based on state space, which makes full use of system states and can be applied to both SISO and MIMO, lower-order and higher-order, minimum phase and nonminimum phase systems. On one hand, EPID retains the simplicity of PID control. It only requires minimum model information, including the state information and the structure information (minimum phase or nonminimum phase). On the other hand, EPID makes full use of the state information and is more specified for different systems, such as SISO/MIMO, lower-order/higher-order, and especially minimum phase/nonminimum phase systems, which are treated differently. For minimum phase systems, the integral gain can be set very high to achieve accurate tracking. But for nonminimum phase systems, the integral gain cannot be set too high to ensure stability, which reflects the performance limitation. The points are illustrated through the hypersonic vehicle and the cart pole examples. In the further, we will do further study and give a more rigorous analysis of this phenomenon.

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