

# Posture Stabilization Control for a Quadruped Robot Walking on Swaying Platforms

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**Abstract**—Compared to wheeled and tracked robots, legged robots like quadruped robots have much more degrees of freedom, which makes them capable to maintain a constant posture on uneven terrains as well as moving platforms. Previous research has focused on the locomotion of quadruped robots in multiple motionless unstructured environments, while the problem of walking on swaying platforms like the shaking cable bridge is rarely studied. The posture stabilization control on swaying surfaces remains an open problem because of the unpredictability of the external forces and torques acting on the feet caused by the movement of the platform. The main work of this article is to present a method that maps static gaits from stationary rigid platforms to swaying rigid platforms while keeping the torso posture level during the whole walking process. The proposed method can greatly suppress the vibration of the fragile items or attached cameras carried by the robot, which is important in situations such as emergency response. Simulation results confirm the effectiveness of the method proposed.

## I. INTRODUCTION

A quadruped robot has distinct advantages when working in unstructured terrains and harsh environments. Because of the many degrees of freedom, the gait planning of quadruped robots is highly flexible, making them capable to accomplish more complex tasks compared to wheeled and tracked vehicles. For example, the BigDog robot from Boston Dynamics can travel on inclines with a variety of surfaces and can jump about 1.1 meters with loads [1]. Its younger fellow SpotMini can easily walk up and down steep stairs. MIT Cheetah 3 is allowed to do stair climbing without external sensors [2]. The Mini Cheetah robot which has a similar design and actuation to MIT Cheetah 3 can execute a 360° backflip serving as an excellent stress test on peak torques and powers [3]. Many other experiments have been carried out so far which validate the flexibility of quadruped robots [4], [5].

Although a large number of terrains have been considered for the gait design of legged robots [6],[7], the walking gaits on swaying platforms are rarely studied. The swaying platforms are widely seen in disaster sites and transportation

vehicles. The cable bridge, the shaking floor of an unstable building, the cabin of an aircraft in flight, the ship deck, and the surface of a moving vehicle are all examples of swaying platforms. The method designed for quadruped robots to walk on such platforms is important for emergency response, fragile items transportation, and stabilization of cameras equipped on the robot.

The walking method for quadruped robots moving on the swaying platform is a challenging problem due to the complexity and unpredictability of the shaking support surface. Unexpected external forces and torques acting on the foot may break the balance of the robot during the walking process so the soft-landing problem [8]-[11] must be considered for this kind of scenario. Although many static gaits are robust enough on the swaying platform because the trajectory of the zero-moment point (ZMP) is well planned based on some of the stability criteria [6],[12]-[16], the torso will wave along with the platform. This phenomenon harms the stable operation of the carried equipment on the quadruped robot. For example, it will be difficult for cameras to focus on their target object.

The continuous-phase dynamic of the quadruped robots on the swaying platform is getting more and more attention and is increasingly studied [17], [18], nevertheless, these researches didn't address the problem of discrete foot-landing walking. Reference [19] creates a control approach to realize provable stable walking on a platform with pitching motion, while the posture of the torso of the quadruped robot is not deliberately controlled.

The main contribution of this article is presenting a posture stabilization controller for quadruped robots walking on swaying rigid platforms by mapping static gaits from stationary rigid platforms to swaying rigid platforms while keeping the torso posture level during the whole walking process without any help of visual sensors. The simulation model is built in CoppeliaSim and the simulation results confirm the effectiveness of the method proposed.

The outline of this article is as follows. First, the problem is formulated. The research background and the main idea of the control method are introduced. The robot model is given and the kinematics of the model is analyzed in Section II. Control algorithm about the attitude adjustment method and the gait planning approach is given next in Section III. The simulation results are shown in Section IV. An intuitive experiment is also done to show the application of this posture stabilization control method. Finally, the conclusion and future work are given in Section V.

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## II. PRELIMINARY

### A. Problem Formulation

The swaying platforms in this paper refer to a series of surface shaking due to the environmental impact. Swaying platforms are commonly seen in the real world and some typical examples as shown in Figure 1. The walking ability of quadruped robots on these kinds of surfaces is essential to some special scenario tasks. Emergency rescue is one of the scenarios where the mobility of vehicles on shaking ground is requested. It will be helpful for salvagers to have a kind of robot which could help with searching and transportation in unstable buildings in an earthquake or on a violent shaking suspension bridge caused by vortex vibration, which may protect humans from being hurt during the rescue. The surface or interior of the transportation such as vehicles, vessels, and aircraft can also be seen as swaying rigid platforms. A posture stabilization method plays an important role in the mission of protecting fragile items, which makes it possible for a quadruped robot to work as an auxiliary targeting system for devices that need to focus on target objects. Thus, posture stabilization control for a quadruped robot walking on a swaying platform is worth studying.



Figure 1. Typical swaying platforms

For quadruped robots walking on a stationary plane, the position and angle of the supporting surface remain unchanged, so that the contact surface of the feet and the platform is a stationary horizontal plane or a plane with a specific tilt angle. When gait planning is performed on such planes, the torso trajectory is relatively stable and the position of the foot point is predictable. When the motion of platforms is introduced, the movement of the platform will impose unknown forces and moments on the end of the feet, so that the gait planned for the stationary plane cannot eliminate the bumps of the torso, as a result, the feet will easily collide with the contact surface undesirably resulting in the instability of the robot.

In our formulation, the scene is relatively simple: the platform is a rigid flat plane that rolls and pitches constantly. The main objective of this paper is to propose an attitude-adjustment-based method that allows the quadruped robots to locomote over such platforms smoothly and keep the attitude of the torso level during its walking process.

An Inertial Measurement Unit (IMU) at the center of gravity (CoG) of the torso measuring roll, pitch, and yaw angles is the only sensor that is needed in the control method proposed, without any vision sensors or force sensors at the joints or end of the feet to help with detecting the state of the platform. Also, the movement of the platform is not given to the controller in advance, as the movement of the platform is always unknown to the robot in the real world.

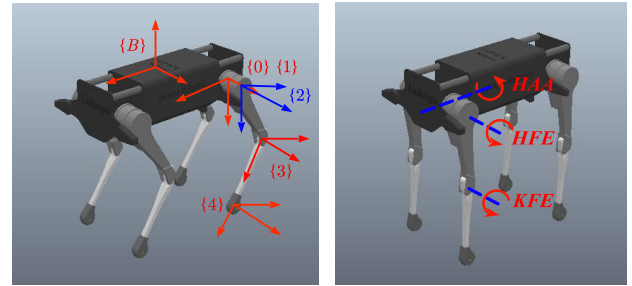
We model a quadruped robot according to the physics parameters of the Laikago robot from UnitreeRobotics. Modified DH (MDH) method is used to model the quadruped robot. Forward kinematics and inverse kinematics are

analyzed to obtain the conversion relationship between joint angles and feet position in the Cartesian coordinate frame. A virtual frame is designed in our methods to help with implementing the gait mapping from static platforms to swaying platforms.

First, the torso attitude is adjusted in real-time based on the IMU information to maintain horizontal and steady. At the same time, the angle of the platform is estimated by the position of the feet based on joint angle information and the kinematic model of the robot. Then, the planned feet trajectory is mapped to the swaying platforms according to the real-time status of the platform. The foot tips will keep at a suitable distance from the platform during its swinging period and land gently at the end of this period. Simulation in CoppeliaSim verifies the effectiveness of the mapping method.

### B. Robot Model

Each of the four legs has three joints including a hip abduction/adduction (HAA) joint, a hip flexion/extension (HFE) joint, and a knee flexion/extension (KFE) joint. These three joints enable fully 3D control of the foot-tip position control. The standing appearance of Laikago is shown in Figure 2(a). Another posture of Laikago is shown in Figure 2(b) when all of the hip and knee joints of each leg are set at  $0^\circ$ .  $\theta_i (i = 1, 2, 3)$  are the angle of the HAA joint, the HFE joint, and the KFE joint respectively. Note that the joints cannot reach the state in Figure 2(b) because of the physical limitation of the joint. This state is only a reference pose for modeling.



(a) Laikago

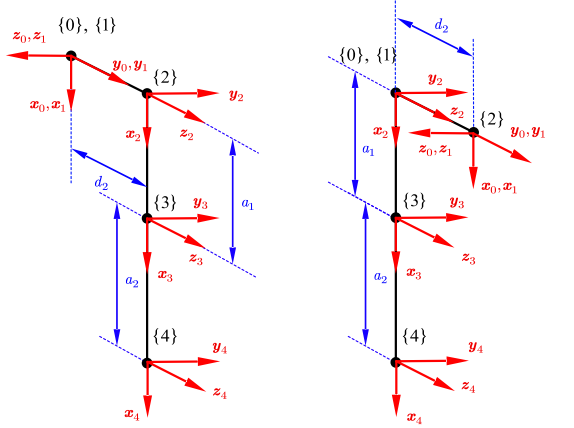
(a) Laikago ( $\theta_i = 0, i = 1, 2, 3$ )

Figure 2. Appearance of Laikago

The Modified Denavit Hartenberg (MDH) convention is used to formulate the quadruped robot model. Since legs on the same side have identical structures, we only need to model one leg of each side (Figure 3 (a) and Figure 3(b)). The DH parameters shown in Figure 3 are listed in TABLE I. Since the mass of the four legs of Laikago is much smaller compared to the mass of the torso, the geometric center of the torso is treated as the center of mass of the quadruped robot approximately as well as the original point of the torso coordinate  $\{B\}$  as shown in Figure 2(a). Different transfer matrices from the torso coordinate frame to the foot coordinate frame are used for each leg, and the transfer matrix for one leg is shown as follow

$${}^B T_4 = \begin{bmatrix} -s_{23} & -c_{23} & 0 & x - a_2 s_2 - a_3 s_{23} \\ s_1 c_{23} & s_1 s_{23} & c_1 & y + d_2 c_1 + a_2 s_1 c_2 + a_3 s_1 c_{23} - s_1 s_2 s_3 \\ -c_1 c_{23} & c_1 s_{23} & s_1 & z + d_2 s_1 - a_2 c_1 c_2 - a_3 c_1 c_{23} - c_1 s_2 s_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where  $s_i$  and  $c_i$  ( $i=1, 2, 3$ ) represents  $\sin(\theta_i)$  and  $\cos(\theta_i)$ .  $s_{ij}$  and  $c_{ij}$  represents  $\sin(\theta_i + \theta_j)$  and  $\cos(\theta_i + \theta_j)$ .  ${}^B \mathbf{p}_0 = [x \ y \ z]^T$  refers to the position of the origin of the HAA joint frame  $\{0\}$  in the torso frame  $\{B\}$ . Subscripts that distinguish the legs are omitted in this article for simplicity.



(a) Left leg MDH coordinate frame (b) Right leg MDH coordinate frame  
Figure 3. MDH coordinate frame of the legs

TABLE I. DH PARAMETERS

Link	DH parameters	Values	Link	DH parameters	Values
1	$a_0$	0	3	$a_2$	0.25
	$\alpha_0$	$0^\circ$		$\alpha_2$	$0^\circ$
	$d_1$	0		$d_3$	0
	$\theta_1$	$\theta_1$		$\theta_3$	$\theta_3$
2	$a_1$	0	4	$a_3$	0.25
	$\alpha_1$	$-90^\circ$		$\alpha_3$	$0^\circ$
	$d_2$	$0.037(Left) / -0.037(Right)$		$d_4$	0
	$\theta_2$	$\theta_2$		$\theta_4$	0

The inverse kinematics of the model are analyzed based on (1). The foot tip position vector with respect to torso frame  $\{B\}$  is defined as  ${}^B \mathbf{p}_4 = (p_x, p_y, p_z)^T$  (the subscript 4 is omitted in the rest of the paper), and the corresponding joint angles can be solved as follow

$$\begin{cases} \theta_1 = \pm \cos^{-1} \left( \frac{d_2}{\sqrt{(p_y - y)^2 + (p_z - z)^2}} \right) + \phi \\ \theta_2 = \pm \cos^{-1} \left( \frac{x - p_x}{\sqrt{(a_2 + a_3 c_3)^2 + a_3^2 s_3^2}} \right) + \phi' \\ \theta_3 = \pm \cos^{-1} \left( \frac{(x - p_x)^2 + (s_1(p_y - y) - c_1(p_z - z))^2 - a_2^2 - a_3^2}{2a_2 a_3} \right) \end{cases} \quad (2)$$

where

$$\phi = \text{atan2}(p_z - z, p_y - y) \quad (3)$$

$$\phi' = \text{atan2}(a_2 + a_3 c_3, a_3 s_3) \quad (4)$$

the value which is closer to the joint angle at the previous moment is taken. This inverse kinematics (IK) model is used in the following controller design.

### III. CONTROLLER DESIGN

#### A. Overall Control Architecture

The overall control architecture of the walking method proposed is shown in Figure 4. The system has three main parts. An attitude adjustment policy is used in the control architecture to stabilize the posture of the torso with the help of the IK solution based on the model of the quadruped robot. Walking gait is planned off-line and mapped to the robot in real-time according to the current posture of the torso.

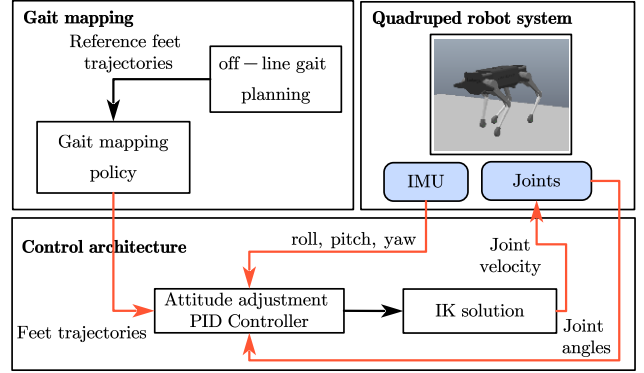


Figure 4. Overall control architecture

#### B. Attitude Adjustment

Attitude Adjustment is carried out in real-time in the proposed method. We first assume that the friction between the platform and the feet is big enough so that the robot would not slip during its walking process. The rotation matrix of the frame  $\{B\}$  relative to the world frame  $\{W\}$  can be written as follow

$${}^w \mathbf{R}_t = \mathbf{R}_x(\alpha) \mathbf{R}_y(\beta) \mathbf{R}_z(\gamma) \quad (5)$$

where  $\mathbf{R}_x$ ,  $\mathbf{R}_y$  and  $\mathbf{R}_z$  are the rotation matrices about the x-axis, y-axis, and z-axis respectively.  $\alpha, \beta, \gamma$  are floating-base roll, pitch, and yaw angles of the torso concerning the frame  $\{W\}$ .

Taking the coordinate frame of the supporting surface of the foot as a reference, the torso posture is controlled by adjusting the angles of the joints to compensate for the fluctuation of the platform. PID controller is used to adjusting the control input  $u_{\alpha i}, u_{\beta i}, u_{\gamma i}$  in (6)

$$\begin{cases} u_{\alpha i} = (1 - K_p) u_{\alpha(i-1)} + K_p \alpha_i + I_{\alpha i} + K_d (\alpha_i - \alpha_{i-1}) \\ u_{\beta i} = (1 - K_p) u_{\beta(i-1)} + K_p \beta_i + I_{\beta i} + K_d (\beta_i - \beta_{i-1}) \\ u_{\gamma i} = (1 - K_p) u_{\gamma(i-1)} + K_p \gamma_i + I_{\gamma i} + K_d (\gamma_i - \gamma_{i-1}) \end{cases} \quad (6)$$

where  $K_p$  is the proportional gain,  $K_d$  is the differential gain and  $I_{\alpha i}, I_{\beta i}, I_{\gamma i}$  are the integral parts with upper and lower bounds. Footmark  $i$  and  $(i-1)$  of the control input and floating-base angles are used to distinguish the signal at the current moment and the signal a time step ago. The target foot tip position vector  ${}^B \mathbf{p}_t$  of each leg can be calculated as (7)

$$\begin{cases} {}^w \mathbf{R}_{ti} = \mathbf{R}_x(u_{\alpha i}) \mathbf{R}_y(u_{\beta i}) \mathbf{R}_z(u_{\gamma i}) \\ {}^B \mathbf{p}_t = {}^w \mathbf{R}_{ti}^{-1} ({}^B \mathbf{p}_c - {}^B \mathbf{p}_0) + {}^B \mathbf{p}_0 \end{cases} \quad (7)$$

where  ${}^B \mathbf{p}_c$  is the vector of the real-time foot tip position calculated through real-time joint angles and forward kinematics model and  ${}^B \mathbf{p}_0$  is the position of the origin of frame  $\{0\}$ . An Intuitive adjustment effect can be seen in Figure 5. By calculating the desired feet position  ${}^B \mathbf{p}_i$  for each leg and taking the IK solution of the joints as control inputs, the robot is able to keep its torso level. Position control is used to adjust the joint angles with a limitation of the maximum joint speed to protect the joint motors.

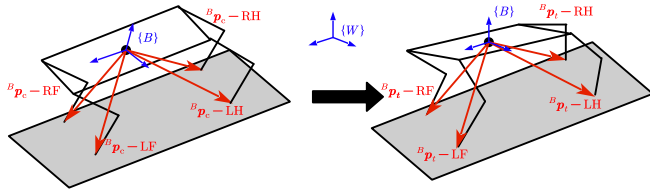


Figure 5. Attitude adjustment

### C. Gait Planning

A basic walking cycle for a quadruped robot walking on a static platform is shown in Figure 6.

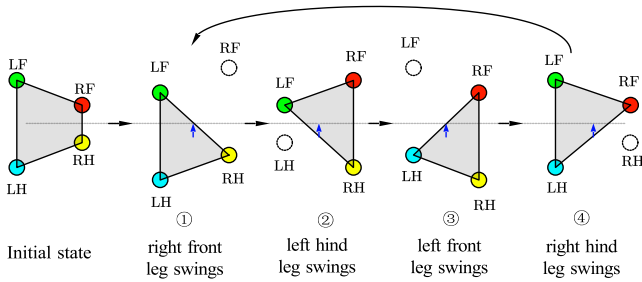
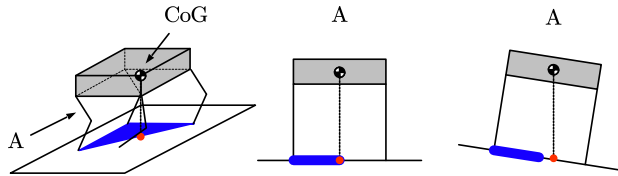


Figure 6. Quadrupedal walking cycle

The blue arrowed line indicates the trajectory of the projection of the center of mass of the robot on the platform. When this kind of gait is used directly on a swaying platform, the projection of CoG can easily fall off the edge of the supporting polygon due to the movement of the platform as shown in Figure 7. If the support legs leave the ground, the swinging leg will collide with the platform when it moves to the originally planned foothold, causing the quadruped robot to shake severely and eventually tip over.



(a) Stationary platform (b) Dynamic platform

Figure 7. Projection of CoG on stationary and swaying platforms

To solve this problem, an attitude-adjustment-based walking method for quadruped robots on swaying rigid platforms is designed, which maps the basic gait from stationary platforms to the swaying platform. The specific steps are as follows:

1) Adjust the body posture of the quadruped robot in real-time to keep it level through the method mentioned in Section III part B.

2) Let the projection of the CoG move to the incentre of the supporting leg triangle before the leg is swinging up.

3) When the leg is swinging, map the planned trajectory to the swaying platform based on the real-time state of the platform.

The mapping method in 3) is a geometric mapping based on a virtual frame  $\{F_v\}$ . The origin of  $\{F_v\}$  is placed at  $(0, 0, -h)$  in frame  $\{B\}$ , where  $h$  is the desired height of the robot. The direction of x-axis, y-axis and z-axis are defined by unit vectors  $\mathbf{n}_x$ ,  $\mathbf{n}_y$  and  $\mathbf{n}_z$  respectively

$$\begin{cases} \mathbf{n}_x = \frac{1}{2} \left( \frac{{}^B \mathbf{p}_{RF} - {}^B \mathbf{p}_{RH}}{|{}^B \mathbf{p}_{RF} - {}^B \mathbf{p}_{RH}|} + \frac{{}^B \mathbf{p}_{LF} - {}^B \mathbf{p}_{LH}}{|{}^B \mathbf{p}_{LF} - {}^B \mathbf{p}_{LH}|} \right) \\ \mathbf{n}_y = \frac{1}{2} \left( \frac{{}^B \mathbf{p}_{RF} - {}^B \mathbf{p}_{LF}}{|{}^B \mathbf{p}_{RF} - {}^B \mathbf{p}_{LF}|} + \frac{{}^B \mathbf{p}_{RH} - {}^B \mathbf{p}_{LH}}{|{}^B \mathbf{p}_{RH} - {}^B \mathbf{p}_{LH}|} \right) \\ \mathbf{n}_z = \mathbf{n}_x \times \mathbf{n}_y \end{cases} \quad (8)$$

where vectors  ${}^B \mathbf{p}_{RF} = (x_{RF}, y_{RF}, z_{RF})^T$ ,  ${}^B \mathbf{p}_{RH} = (x_{RH}, y_{RH}, z_{RH})^T$ ,  ${}^B \mathbf{p}_{LF} = (x_{LF}, y_{LF}, z_{LF})^T$ , and  ${}^B \mathbf{p}_{LH} = (x_{LH}, y_{LH}, z_{LH})^T$  are the real-time position of the foot tips of the right front leg, right hind leg, left front leg, and left hind leg respectively. When one of the legs is swinging, taking the situation that the right front leg is swinging for example,  $\mathbf{n}_x$ ,  $\mathbf{n}_y$  and  $\mathbf{n}_z$  are defined as follow

$$\begin{cases} \mathbf{n}_x = \frac{{}^B \mathbf{p}_{LF} - {}^B \mathbf{p}_{LH}}{|{}^B \mathbf{p}_{LF} - {}^B \mathbf{p}_{LH}|} \\ \mathbf{n}_y = \frac{{}^B \mathbf{p}_{LH} - {}^B \mathbf{p}_{RH}}{|{}^B \mathbf{p}_{LH} - {}^B \mathbf{p}_{RH}|} \\ \mathbf{n}_z = \mathbf{n}_x \times \mathbf{n}_y \end{cases} \quad (9)$$

A height vector  $\mathbf{h} = (0, 0, -h)^T$  is used to record the height of the quadruped robot. For any static gait planned for walking on a stationary platform, the target foot tip position  ${}^B \mathbf{p}_i$  can be written in the form as follow

$${}^B \mathbf{p}_i = \mathbf{h} + l_x \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + l_y \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + l_z \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (10)$$

where  $l_x$ ,  $l_y$  and  $l_z$  are coefficients of the unit vectors of axes of  $\{B\}$ . Then we can get a mapped target foot tip position  ${}^B \mathbf{p}_i'$  for the swaying platform which can be calculated as follow

$${}^B \mathbf{p}_i' = \mathbf{h} + l_x \mathbf{n}_x + l_y \mathbf{n}_y + l_z \mathbf{n}_z \quad (11)$$

With the help of the geometry mapping, a position-controlled gait designed for a stationary platform can be used on a swaying platform by replacing the origin target foot tip position with the new target  ${}^B \mathbf{p}_i'$ . As the unit vector  $\mathbf{n}_x$ ,  $\mathbf{n}_y$  and  $\mathbf{n}_z$  are computed in real-time, the new target is well-matched for the swaying platform at any time. In this way, the distance between the foot tip and the platform is almost the same as that when walking on the static platform, as a result, there will not be a strong collision when the foot touches the swaying platform. In our experiment, a simple cycloid trajectory is used and mapped in real-time.



#### IV. SIMULATION RESULTS

The CoppeliaSim simulation software is used to validate the effectiveness of the method proposed in this article. Dynamic engine Newton is used and the simulation time step is set as 50ms. The physical parameters of the quadruped robot in the simulation are set according to the official manual of Laikago. The swaying rigid platform shakes with a whole-body rolling motion (Nominal rolling amplitude =  $\pm 4^\circ$ . Nominal rolling frequency = 0.125Hz), and a whole-body pitching motion (Nominal pitching amplitude =  $\pm 4^\circ$ . Nominal rolling frequency = 0.11Hz). The frequency of rolling and pitch motion is set a little differently to mimic random platform motion. The simulation scene is shown in Figure 8.

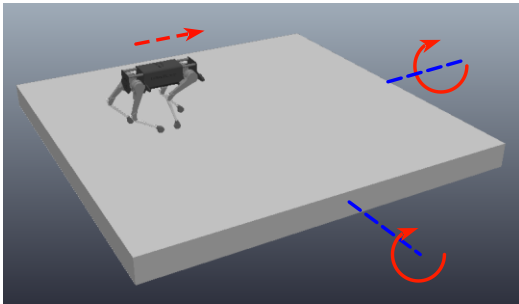


Figure 8. Simulation model

The gait parameters are as follows

- 1) Step length = 30cm.
- 2) Maximum swing-foot height = 4cm.
- 3) Walking speed = 0.9m/min.

A fifty-second simulation is taken to let the quadruped robot walk straight on the swaying platform and the result is shown in Figure 9.

It can be seen from the simulation result shown in Figure 9 that the quadruped robot can walk on the swaying platform with the roll angle and pitch angle of the torso within  $1^\circ$ . Simulation results verify the effectiveness of the proposed method. It shows that the walking of a quadruped robot on the moving platform surface can be realized by adjusting the torso posture in real-time and mapping the static platform gait to the moving platform.

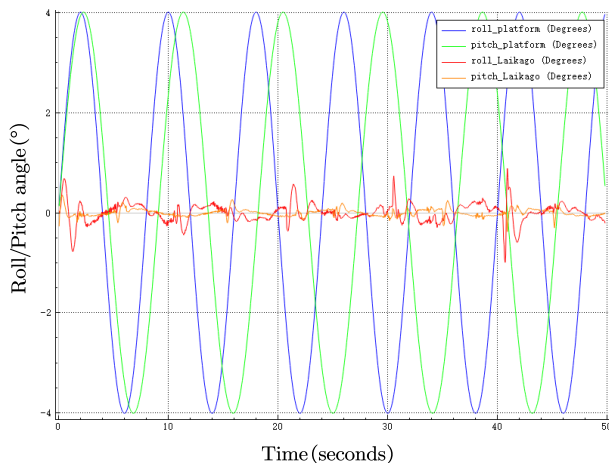


Figure 9. Simulation result of walking on a swaying platform

Another simulation is also performed to test the effectiveness of the mapping method alone. In this case, we didn't move the projection of the CoG to the incentre of the supporting leg triangle before the leg is swinging up and the CoG keeps moving forward. The walking speed goes up to 1.8m/min because there's no CoG moving period. The red dotted lines in Figure 10 show the amplitude peak of the roll and pitch angle in the previous experimen. In this simulation, the amplitude of the torso becomes a little bigger (roll and pitch angle within  $1.75^\circ$ ) as a result. Both of the simulations show the effectiveness of the mapping method.

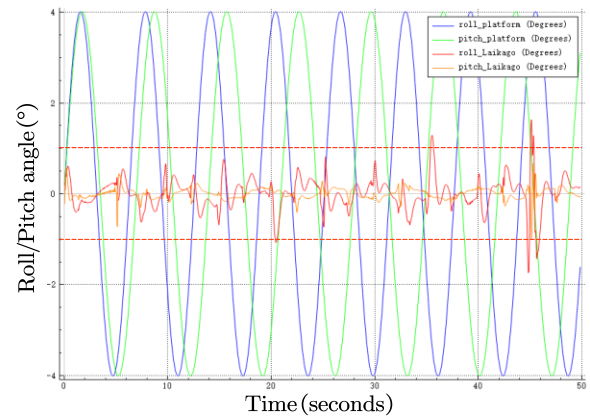


Figure 10. Comparison simulation result

An intuitive simulation was taken, during which a camera was put on the torso of the quadruped robot to test the ability to stabilize the camera view. The simulation scenario is shown in Figure 11. A cube with a side length of 0.5 m is located 20m away in front of the robot.

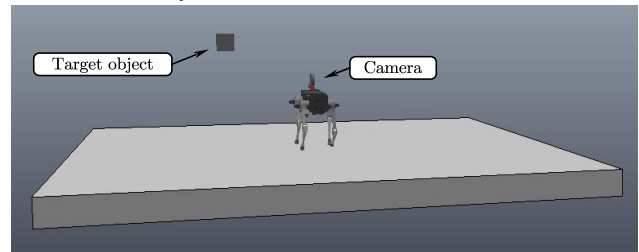
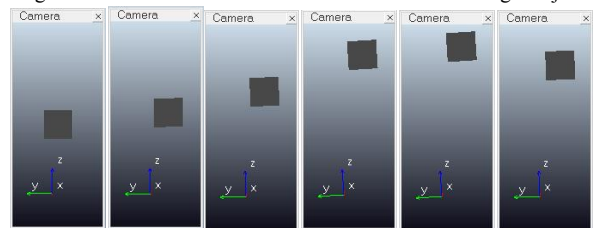
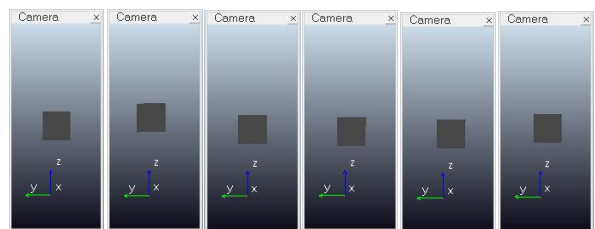


Figure 11. The simulation scene with a camera and a target object



(a) Camera view of a static gait



(b) Camera view of the gait using posture stabilization control

Figure 12. Camera views comparison

The comparison of the camera views of a quadruped robot walking with a normal static gait and walking with the gait using the posture stabilization control method proposed is given in Figure 12(a) and Figure 12(b) respectively. It can be seen that the cube floats up and down when nothing is done to keep the posture level. However, the target object remains in the center of view during the walking process when the posture of the torso is stabilized.

## V. CONCLUSION

In this article, a posture stabilization control method for a quadruped robot walking on a swaying platform is presented. The torso posture of the quadruped robot is adjusted in real-time to maintain level during the whole walking process. Static gait planned for robots walking on stationary platforms is mapped to the swaying platforms to avoid strong collision between the swing leg and the platform with the help of a virtual frame. The proposed method improves the stability of the quadruped robot on the moving support surface and can provide a relatively stable working environment for the equipment mounted on the torso. CoppeliaSim simulation validates the effectiveness of the proposed method.

Though walking successfully, the walking speed of the robot is limited and, in the future, more efficient compliant control will be developed to tune the landing foot position as apparent vibration often occurs when the swinging foot touches the platform. Meanwhile, only the current information of the platform is used in this method, a prediction of platform movement can be made to help with the planning of foot trajectories so that a faster walking speed is achievable. A temporal Convolutional Network and reinforcement learning based on state information of the robot may help the system to show better performance.

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