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Control for MIMO Systems with No Relative Degree: Output Redefinition versus Dynamic Extension

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Abstract— **Output redefinition-based dynamic inversion (ORDI) control is applied to solve the control problem for multi-input-multi-output (MIMO) systems with no relative degree and is compared with the traditional dynamic extension-based dynamic inversion (DEDI) control. A MIMO system has no relative degree if its control matrix is singular, preventing the direct use of the powerful nonlinear control method, dynamic inversion. For this problem, dynamic extension is a traditional solution, which makes dynamic extension at the input side to achieve a relative degree. DEDI results in a fully linearized system of higher order. But the requirement to calculate the higher order derivatives of the output makes it difficult to apply to complex systems. ORDI provides a new solution for the existing problem. It achieves a relative degree by redefinition of a new output, leading to a partially linearized system cascaded with stable zero dynamics. ORDI is much easier to implement for complex systems and reduces the computational burden, though it has some performance limitations. A linear system example along with the application to a hypersonic flight vehicle are provided to illustrate the concept of ORDI and its differences with DEDI.**

I. INTRODUCTION

Dynamic inversion, which is also called feedback linearization [1], is a powerful nonlinear control method. However, the precondition to use dynamic inversion is that the system has a well-defined relative degree. Consider the following multi-input-multi-output (MIMO) system

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} \\ \mathbf{y} &= \mathbf{h}(\mathbf{x}) \end{aligned} \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n, \mathbf{u}, \mathbf{y} \in \mathbb{R}^m$, $\mathbf{g}(\mathbf{x}) = [g_1(\mathbf{x}), g_2(\mathbf{x}), \cdots, g_m(\mathbf{x})]^T$, and $\mathbf{h}(\mathbf{x}) = [h_1(\mathbf{x}), h_2(\mathbf{x}) \cdots, h_m(\mathbf{x})]^T$. According to [1], the system is said to have a relative degree $\{r_1, ..., r_m\}$ at a point x^o if

(i) $L_{g_j} L_f^k h_i(x) = 0$ (L represents Lie derivative) for all $1 \le j \le m$, for all $1 \le i \le m$ for all $k < r_i - 1$ and for all x in the neighborhood of x^o.

(ii) the m×m control matrix

$$\mathbf{A} = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1(x) & \cdots & L_{g_m} L_f^{r_1-1} h_1(x) \\ L_{g_1} L_f^{r_2-1} h_2(x) & \cdots & L_{g_m} L_f^{r_2-1} h_2(x) \\ \cdots & \cdots & \cdots \\ L_{g_1} L_f^{r_m-1} h_m(x) & \cdots & L_{g_m} L_f^{r_m-1} h_m(x) \end{bmatrix}$$

is nonsingular at $x = x^o$.

The system has no relative degree if the matrix $\mathbf{A}$ is singular. Therefore, dynamic inversion can't be used since the inversion of $\mathbf{A}$ doesn't exist. To solve this problem, the dynamic extension method is proposed in [1,2], where the input is viewed as a state of an extended dynamic system affected by a new input to achieve a relative degree. This method has been applied to many practical systems, such as aviation aircraft [1], quadrotor [3,4], car-like robot [5], manipulators [6,7], and hypersonic vehicle [8-10], just name a few. However, the dynamic extension method may introduce a significant complication in the design of the control law for the extended system.

From the definition of relative degree [1], it can be seen that it is associated with the input-output map. Therefore the relative degree will change if the input or output is redefined. Dynamic extension changes the input-output map by redefinition of a new input. The other way to change the input-output map is to change the output. Thus it naturally leads to the concept of output redefinition.

Output redefinition [11] is a traditional method applied to nonminimum phase systems. A nonminimum phase system refers to a system with unstable zero dynamics [1]. If a system has a relative degree smaller than the system order, there will be some remaining dynamics that can't be included in the input-output map, which are called zero dynamics. Output redefinition can change the input-output map and thus obtain stable zero dynamics. Many methods are proposed to find a minimum phase output, such as the method through B-I norm form [11], the flatness-based approach [12] and the synthetic output method [13, 14]. Most recently, output redefinition-based dynamic inversion (ORDI) is presented in our previous work [15] to solve the control problem of nonminimum phase systems in a systematic way. In this paper, we will show that ORDI is also applicable to control MIMO systems with no relative degree, thus broaden the application range of ORDI.

A detailed procedure is given on how to apply ORDI to MIMO systems with no relative degree. First, a new output is constructed to obtain a system that has a well-defined relative degree, and then dynamic inversion can be applied to the new output. The new output is constructed in a simple way, which is a linear combination of system states. The relative degree of

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the new output is smaller than the system order, so there will be zero dynamics. To guarantee the zero dynamics be stable, the root locus approach is utilized to determine the feasible value for the combination coefficient.

ORDI gives a new option to the control issue for MIMO systems with no relative degree. Compared to dynamic extension-based dynamic inversion (DEDI), some significant differences are shown: (1) DEDI achieves a relative degree by changing the input while ORDI achieves a relative degree by changing the output; (2) DEDI obtains a dynamic control law while ORDI obtains a static control law; (3) DEDI results in a system of higher order while ORDI results in a lower order system cascaded with a stable zero dynamics; (4) DEDI requires to calculate higher-order derivatives of the output, making it difficult for application to complex system. In comparison, ORDI is much easier to carry out and involves less computational burden; (5) DEDI is fully linearization, but ORDI is partially linearization and has some performance limitations when applied to nonlinear system.

II. A LINEAR SYSTEM EXAMPLE

Consider a simple linear system as follows:

$$\dot{x}_1 = u_1,\ \dot{x}_2 = x_3 + u_1,\ \dot{x}_3 = u_1 + u_2 \tag{2}$$

where u_1, u_2 are control inputs and x_1, x_2, x_3 are system states. The system outputs are $y_1 = x_1, y_2 = x_2$. The system has no relative degree since the derivatives of y_1 and y_2 are affected both by u_1 and none by u_2. The input-output dynamics can be written as follows

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{3}$$

Since the control matrix in (3) is singular, dynamic inversion can't be directly applied to this system.

A. Dynamic Extension-Based Dynamic Inversion Control

To achieve a relative degree, dynamic extension can be made on the input side. For system (2), the dynamic extension can be done as follows:

$$\dot{u}_1 = v_1 \tag{4}$$

where v_1 is treated as a new input. With this definition, u_1 becomes a system state. Suppose the output commands are x_1^*, x_2^*, respectively, and denote the regulated outputs as $e_1 = x_1 - x_1^*$, $e_2 = x_2 - x_2^*$. Following the usual procedure, the outputs are differentiated until the input appears

$$\dot{e}_1 = u_1,\ \ddot{e}_1 = v_1;\ \dot{e}_2 = x_3 + u_1,\ \ddot{e}_2 = u_1 + u_2 + v_1 \tag{5}$$

Denote $v_2 = u_1 + u_2 + v_1$, it follows that

$$\ddot{e}_1 = v_1,\ \ddot{e}_2 = v_2 \tag{6}$$

Therefore the relative degree is $\{2,2\}$ now. The original system is fully linearized into system (6). Design the inputs as follows

$$v_1 = -k_{11}\dot{e}_1 - k_{12}e_1,\ v_2 = -k_{21}\dot{e}_2 - k_{22}e_2 \tag{7}$$

where $k_{11}, k_{12}, k_{21}, k_{22}$ are positive control gains. Then the closed-loop system becomes

$$\ddot{e}_1 = -k_{11}\dot{e}_1 - k_{12}e_1,\ \ddot{e}_2 = -k_{21}\dot{e}_2 - k_{22}e_2 \tag{8}$$

Therefore, by virtue of dynamic extension, the original system of order 3 is fully linearized into a 4th-order linear system which is asymptotically stable.

B. Output Redefinition-Based Dynamic Inversion Control

Recall system (2), $\dot{x}_1, \dot{x}_2$ are affected only by u_1 while $\dot{x}_3$ is affected by both u_1 and u_2. With the output x_1 unchanged, the other output x_2 can be combined with x_3 to construct a new output affected by both u_1 and u_2. Take into consideration the command, the new output is designed as $e_3 = x_3 + \lambda_1 e_2$, where λ_1 is a positive coefficient to be designed. Taking the derivative of e_3 yields

$$\dot{e}_3 = \dot{x}_3 + \lambda_1 \dot{x}_2 = \lambda_1 x_3 + (\lambda_1 + 1)u_1 + u_2 \tag{9}$$

With the new output $[e_1, e_3]^T$, the input-output dynamics becomes

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ \lambda_1 x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ \lambda_1 + 1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{10}$$

The control matrix is nonsingular now and the relative degree of the new output is $\{1,1\}$. Design the control inputs as

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \lambda_1 + 1 & 1 \end{bmatrix}^{-1} \left\{ \begin{bmatrix} -k_1 e_1 \\ -k_2 e_3 \end{bmatrix} - \begin{bmatrix} 0 \\ \lambda_1 x_3 \end{bmatrix} \right\} \tag{11}$$

The closed-loop system becomes

$$\dot{e}_1 = -k_1 e_1,\ \dot{e}_3 = -k_2 e_3 \tag{12}$$

where k_1, k_2 are positive control gains.

Now, since the relative degree $1+1=2$ is smaller than the system order 3, there will be a 1st-order zero dynamics remaining. When the new output $[e_1, e_3]^T$ is driven to zero, it follows that $x_3 = -\lambda_1 e_2$ and $u_1 = 0$. Substituting them into $\dot{e}_2$ yields the zero dynamics:

$$\dot{e}_2 = -\lambda_1 e_2 \tag{13}$$

Therefore even though the output e_2 is altered to e_3, the tracking error e_2 can still converge to zero due to the stable zero dynamics (13).

C. Simulation

In the simulation, the initial values of the states are assumed in the origin. The control parameters are selected as $k_{11} = k_{12} = k_{21} = k_{22} = 4$ for DEDI and $\lambda_1 = k_1 = k_2 = 2$ for ORDI, respectively. The commands are given as $x_1^* = x_2^* = 1$. Figure 1 shows the simulation results of DEDI and Figure 2 shows the simulation results of ORDI. It can be seen that both methods achieve good tracking performance.

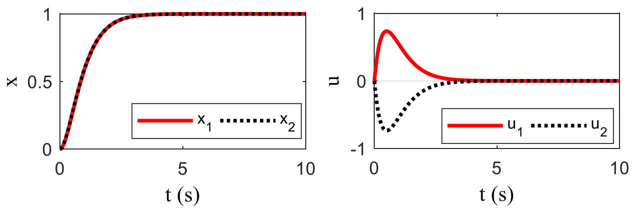


Figure 1. Simulation results of DEDI

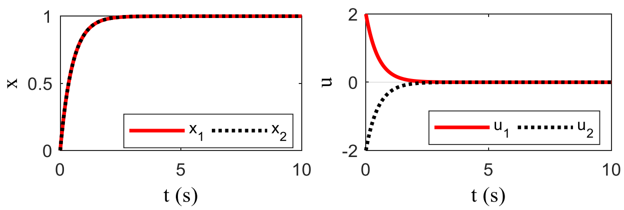


Figure 2. Simulation results of ORDI

From this example, we can see the differences between DEDI and ORDI. They achieve a relative degree from two ways: DEDI by integral extension of a new input and ORDI by redefinition of a new output. As a result, DEDI leads to a dynamic control law and a higher-order closed-loop system, while ORDI results in a static control law and a lower-order closed-loop system cascaded with a zero dynamics system. In the next section, we will derive the ORDI control method for general MIMO systems with no relative degree.

III. ORDI Control for MIMO Systems with No Relative Degree

As shown in [1], if a system has a relative degree which equals to the system order, then all the states can be expressed by the outputs and their derivatives. However, for a MIMO system with no relative degree, there will be some states that cannot be expressed by the outputs and their derivatives, which will be called by internal variable in this paper. It should be noted that internal variable is in the original coordinate and is different from the concept of internal state [1]. The ORDI control scheme includes two steps as shown in Figure 3.

Step 1: Output redefinition

In this step, a linear combination of the original outputs and the internal variables is constructed as a new output to obtain a relative degree and moreover to obtain stable zero dynamics.

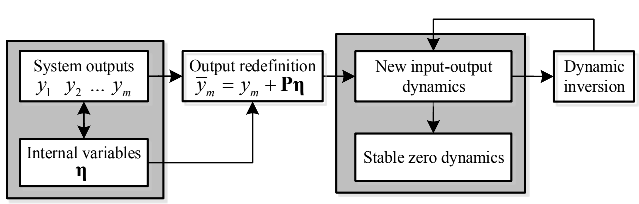


Figure 3. Schematic diagram of ORDI

For linear system with m outputs $y_i, i = 1,\dots,m$, by taking derivatives of each output until the input appears, with the derivation orders $r_i, i = 1,\dots,m$, the input-output dynamics can be written as

$$\begin{bmatrix} y_1^{(r_1)} & y_2^{(r_2)} & \dots & y_m^{(r_m)} \end{bmatrix}^T = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{14}$$

where $\mathbf{u} = [u_1, u_2, \dots, u_m]^T$ is the input vector, $\mathbf{x} = [\boldsymbol{\xi}, \boldsymbol{\eta}]^{\mathrm{T}}$ is the state vector. $\boldsymbol{\xi}$ is the external state vector consists of the outputs and their derivatives; $\boldsymbol{\eta}$ is the internal variable vector, with the internal dynamics

$$\dot{\boldsymbol{\eta}} = \mathbf{A}_{\eta}\mathbf{x} + \mathbf{B}_{\eta}\mathbf{u} \tag{15}$$

It is assumed that $rank(\mathbf{B}) < m$ so the system has no relative degree. To achieve a relative degree, we perform output redefinition by replacing one of the outputs, for instance, the last one by

$$\bar{y}_m = y_m + \mathbf{P}\boldsymbol{\eta} \tag{16}$$

where $\mathbf{P}$ is a vector to be designed. Then the new input-output dynamics becomes

$$\begin{bmatrix} y_1^{(r_1)} & y_2^{(r_2)} & \dots & \bar{y}_m^{(\bar{r}_m)} \end{bmatrix}^T = \bar{\mathbf{A}}\mathbf{x} + \bar{\mathbf{B}}\mathbf{u} \tag{17}$$

The selection of $\mathbf{P}$ should meet two requirements:

1) The new output vector has a relative degree, i.e. $rank(\bar{\mathbf{B}}) = m$;

2) The following zero dynamics are stable:

$$\dot{\boldsymbol{\eta}} = \left(\mathbf{A}_{\eta} - \mathbf{B}_{\eta}\bar{\mathbf{B}}^{-1}\mathbf{A}\right)[\mathbf{0}, \boldsymbol{\eta}]^{\mathrm{T}} \tag{18}$$

Similarly, for the nonlinear system

$$\begin{aligned} \begin{bmatrix} y_1^{(r_1)} & y_2^{(r_2)} & \dots & y_m^{(r_m)} \end{bmatrix}^T &= \mathbf{F}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\mathbf{u} \\ \dot{\boldsymbol{\eta}} &= \mathbf{F}_{\eta}(\mathbf{x}) + \mathbf{G}_{\eta}(\mathbf{x})\mathbf{u} \end{aligned} \tag{19}$$

Assume $rank[\mathbf{G}(\mathbf{x})] < m$. Replacing one of the outputs by

$$\bar{y}_m = y_m + \mathbf{P}\boldsymbol{\eta} \tag{20}$$

Then the new input-output dynamics becomes

$$\begin{bmatrix} y_1^{(r_1)} & y_2^{(r_2)} & \dots & \bar{y}_m^{(r_m)} \end{bmatrix}^T = \bar{\mathbf{F}}(\mathbf{x}) + \bar{\mathbf{G}}(\mathbf{x})\mathbf{u} \tag{21}$$

The selection of $\mathbf{P}$ should also meet two requirements:

1) The new output vector has a relative degree, i.e. $rank[\bar{\mathbf{G}}(\mathbf{x})] = m$;

2) The following zero dynamics are locally stable:

$$\dot{\boldsymbol{\eta}} = \mathbf{F}_{\eta}(\mathbf{0}, \boldsymbol{\eta}) - \mathbf{G}_{\eta}(\mathbf{0}, \boldsymbol{\eta})\bar{\mathbf{G}}(\mathbf{0}, \boldsymbol{\eta})^{-1}\bar{\mathbf{F}}(\mathbf{0}, \boldsymbol{\eta}) \tag{22}$$

To meet the two requirements, we can design the vector $\mathbf{P}$ by root locus method. Through this step, the no relative degree problem is solved and meanwhile a stable zero dynamics is obtained, which forms the foundation of the second step.

Step 2: Dynamic inversion

In this step, dynamic inversion control is performed on the new input-output dynamics for linearization and achieving the desired control objective. Since the new control output vector has a well-defined relative degree, dynamic inversion control can be easily applied to achieve output tracking. What's more, since the zero dynamics is rendered stable in the first step, the overall system is guaranteed to be stable.

From the procedure above, it can be observed that ORDI method has some properties very different from the traditional DEDI method, as shown in Table I.

TABLE I. COMPARISON OF ORDI AND DEDI

	ORDI	DEDI
Way to Achieve a Relative Degree	Output redefinition (Change output)	Dynamic extension (Change input)
Control Law	Static	Dynamic
Closed-loop System	Lowe-order linear system with stable zero dynamics	Higher-order linear system
Computational Burden	Low	High
Linearization Level	Partially linearization	Fully linearization

In the next section, a nonlinear example will be used to further illustrate the implementation of ORDI method and compare it with the DEDI method.

IV. APPLICATION TO A HYPERSONIC FLIGHT VEHICLE

In [15], ORDI is applied to an air-breathing hypersonic vehicle model, which exhibits nonminimum phase behavior. In this paper, we will apply ORDI to another hypersonic flight vehicle model [8] which has a different configuration, and, rather than be nonminimum phase, it has no relative degree. Following [8], the longitudinal dynamics of the hypersonic flight vehicle described by a set of differential equations for velocity V, flight-path angle γ, altitude h, pitch angle θ, and pitch rate q are as follows

$$\begin{aligned} &\dot{V} = \left(T\cos\alpha - D\right)/m - \mu\sin\gamma/r^2 \\ &\dot{h} = V\sin\gamma \\ &\dot{\gamma} = \left(L + T\sin\alpha\right)/\left(mV\right) - \left(\mu - V^2 r\right)\cos\gamma/\left(Vr^2\right) \\ &\dot{\theta} = q,\ \dot{q} = M_{yy}/I_{yy} \end{aligned} \tag{23}$$

The expressions of the thrust T, the lift L, the drag D, and the pitching moment M_{yy} are given by

$$\begin{aligned} &L = \bar{q}SC_L, D = \bar{q}SC_D, T = \bar{q}SC_T \\ &M_{yy} = \bar{q}S\bar{c}\left[C_M\left(\alpha\right) + C_M\left(\delta_e\right) + C_M\left(q\right)\right] \end{aligned} \tag{24}$$

where

$$\begin{aligned} &C_L = 0.6203\alpha, C_D = 0.6450\alpha^2 + 0.004338\alpha + 0.00377 \\ &C_T = \begin{cases} 0.02576\beta & if \quad \beta < 1 \\ 0.0224 + 0.00336\beta & if \quad \beta > 1 \end{cases} \\ &C_M\left(\alpha\right) = -0.035\alpha^2 + 0.036617\alpha + 5.3261\times 10^{-6} \\ &C_M\left(q\right) = \left(\bar{c}/2V\right)q\left(-6.796\alpha^2 + 0.3015\alpha - 0.2289\right) \\ &C_M\left(\delta_e\right) = c_e\left(\delta_e - \alpha\right) \end{aligned} \tag{25}$$

In this study, $C_T = c_\beta \beta$ with $c_\beta = 0.02576$ is taken for convenience.

The control inputs are the throttle setting β and the elevator deflection δ_e. The outputs are the velocity V and the altitude h. The commanded desired values of velocity and altitude are denoted by V^* and h^*, respectively. The objective is to let the tracking errors $e_V = V - V^*$, $e_h = h - h^*$ converge to zero asymptotically.

To begin with, we will first examine the relative degree of this system. System (23) can be rewritten in an affine form:

$$\begin{aligned} &\dot{V} = f_V + g_V\beta \\ &\dot{h} = V\sin\gamma, \dot{\gamma} = f_\gamma + g_\gamma\beta \\ &\dot{\theta} = q, \dot{q} = f_q + g_q\delta_e \end{aligned} \tag{26}$$

where

$$\begin{aligned} &f_V = -D/m - \mu\sin\gamma/r^2, \ g_V = \bar{q}Sc_\beta\cos\alpha/m \\ &f_\gamma = L/\left(mV\right) - \left(\mu - V^2 r\right)\cos\gamma/\left(Vr^2\right) \\ &g_\gamma = \bar{q}Sc_\beta\sin\alpha/\left(mV\right) \\ &f_q = \bar{q}S\bar{c}\left[C_M\left(\alpha\right) - c_e\alpha + C_M\left(q\right)\right]/I_{yy} \\ &g_q = \bar{q}S\bar{c}c_e/I_{yy} \end{aligned} \tag{27}$$

In order to find out the relative degree of the system, take the derivative of the outputs until the input appears:

$$\dot{h} = V\sin\gamma, \ddot{h} = \dot{V}\sin\gamma + V\dot{\gamma}\cos\gamma = f_h + g_h\beta \tag{28}$$

where

$$f_h = f_V\sin\gamma + f_\gamma V\cos\gamma, g_h = g_V\sin\gamma + g_\gamma V\cos\gamma \tag{29}$$

From (26) and (28), it can be seen that the outputs V and h are affected both by β and none by δ_e. Therefore the system has no relative degree.

A. Dynamic Extension-Based Dynamic Inversion Control

To achieve a relative degree, a second-order dynamic extension is appended to the input β

$$\ddot{\beta} = -2\zeta\omega_n\dot{\beta} - \omega_n^2\beta + \omega_n^2\beta_c \tag{30}$$

where ζ, ω_n are parameters to be designed (can also use $\ddot{\beta} = \beta_c$). The commanded value β_c is viewed as the new input. Then the outputs $\left[e_V, e_h\right]^T$ are differentiated until the input appears

$$\begin{bmatrix} e_V^{(3)} \\ e_h^{(4)} \end{bmatrix} = \begin{bmatrix} a_V \\ a_h \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\begin{bmatrix} \beta_c \\ \delta_e \end{bmatrix} \tag{31}$$

The expressions of $a_V, a_h, b_{11}, b_{12}, b_{21}, b_{22}$ are given in [8]. From (31) it can be seen that the relative degree of the system becomes $\{3,4\}$. The dynamic inversion controller is designed as

$$\begin{bmatrix}\beta_c\\ \delta_e\end{bmatrix}=\begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{bmatrix}^{-1}\begin{bmatrix}-a_V-k_{a11}\ddot{e}_V-k_{a12}\dot{e}_V-k_{a13}e_V\\ -a_h-k_{a21}\dddot{e}_h-k_{a22}\ddot{e}_h-k_{a23}\dot{e}_h-k_{a24}e_h\end{bmatrix} \quad (32)$$

where $k_{a11},k_{a12},k_{a13},k_{a21},k_{a22},k_{a23},k_{a24}$ are positive gains to be designed. The closed-loop system is

$$\begin{aligned} e_V^{(3)} &= -k_{a11}\ddot{e}_V-k_{a12}\dot{e}_V-k_{a13}e_V \\ e_h^{(4)} &= -k_{a21}\dddot{e}_h-k_{a22}\ddot{e}_h-k_{a23}\dot{e}_h-k_{a24}e_h \end{aligned} \quad (33)$$

Obviously, the design process above is very difficult due to the calculation of the higher-order derivatives of the outputs. Besides, the control law obtained by DEDI is also very complex which will be computational burdensome.

B. Output Redefinition-Based Dynamic Inversion Control

To apply ORDI to the hypersonic vehicle, similar to the linear example, a new output is constructed as a linear combination of the system states. Recall the reason why the system has no relative degree is that $\dot{V}$ and $\ddot{h}$ are affected both by β and none by δ_e. Take into consideration that $\ddot{\theta}$ is affected by δ_e, thus h and θ can be combined to construct a new output so that its second-order derivative is affected by both β and δ_e.

If the new output is chosen as $e_h+\lambda\theta$, then as $e_h+\lambda\theta$ is driven to zero, e_h will not converge to zero since the equilibrium of θ is nonzero. In order to make e_h converge to zero, the new output can be chosen as $e_h+\lambda e_\theta$, where $e_\theta=\theta-\theta^*$ with θ^* being the equilibrium of θ. When V^*,h^* are given, then θ^* can be obtained by solving (23) with all the states derivatives being zero.

Therefore the new output is selected as $\mathbf{y_c}=[e_V,e_h+\lambda e_\theta]^T$ where λ is a parameter to be designed. Denote $e_c=e_h+\lambda e_\theta$. The input-output dynamics corresponding to the new output are

$$\begin{bmatrix}\dot{e}_V\\ \ddot{e}_c\end{bmatrix}=\begin{bmatrix}f_V\\ f_c\end{bmatrix}+\begin{bmatrix}g_V & 0\\ g_{c\beta} & g_{c\delta}\end{bmatrix}\begin{bmatrix}\beta\\ \delta_e\end{bmatrix} \quad (34)$$

where

$$\begin{aligned} f_c &= f_V\sin\gamma+f_\gamma V\cos\gamma+\lambda f_q \\ g_{c\beta} &= g_V\sin\gamma+g_\gamma V\cos\gamma \\ g_{c\delta} &= \lambda g_q \end{aligned} \quad (35)$$

The relative degree is $\{1,2\}$ now, which means there is a second order (5-1-2 = 2) zero dynamics in this system. To obtain stable zero dynamics, a root locus method will be used to determine the coefficient λ.

For system (34), the inputs to keep zero outputs are

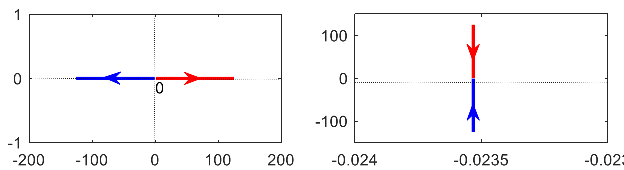
$$\begin{bmatrix}\beta^0\\ \delta_e^0\end{bmatrix}=\begin{bmatrix}g_V & 0\\ g_{c\beta} & g_{c\delta}\end{bmatrix}^{-1}\begin{bmatrix}-f_V\\ -f_c\end{bmatrix} \quad (36)$$

By substituting (36) into the h,γ dynamics, the modified zero dynamics are as follows

$$\dot{e}_h=V\sin\gamma,\ \dot{\gamma}=f_\gamma+g_\gamma\beta^0 \quad (37)$$

By inspection of (37), it can be observed that the combination coefficient λ will affect the zero dynamics through β^0. To guarantee the zero dynamics be stable, the root locus approach is utilized to determine the feasible value of λ.

Figure 4 shows the root locus of the linearized zero dynamics at equilibrium $[15060,110000,0,0.0312,0]^T$. It can be observed that when λ ranges from -1000 to -1, there are two real eigenvalues (one positive, one negative) which go away from the origin as λ increases; when λ ranges from 1 to 1000, there are two LHP complex eigenvalues. Their real parts remain at about -0.0235 and they go toward the real axis as λ increases. Therefore, in order to obtain stable zero dynamics, λ should be a positive value.



(a) λ from -1000 to -1 (b) λ from 1 to 1000

Figure 4. Root locus of linearized zero dynamics

After λ is determined, a dynamic inversion controller can be designed. According to (34), design the control inputs as follows

$$\begin{bmatrix}\beta\\ \delta_e\end{bmatrix}=\begin{bmatrix}g_V & 0\\ g_{c\beta} & g_{c\delta}\end{bmatrix}^{-1}\begin{bmatrix}-k_{c11}e_V-f_V\\ -k_{c21}\dot{e}_c-k_{c22}e_c-f_c\end{bmatrix} \quad (38)$$

where k_{c11},k_{c21},k_{c22} are positive gains to be designed. Then the closed-loop system becomes

$$\dot{e}_V=-k_{c11}e_V,\ \ddot{e}_c=-k_{c21}\dot{e}_c-k_{c22}e_c \quad (39)$$

By selecting proper control gains, (39) can be made asymptotically stable with desired convergence rate. When the new outputs $\mathbf{y_c}=[e_V,e_c]^T$ are driven to zero, the altitude tracking error e_h will also converge to zero under the effect of the modified zero dynamics (37).

From this nonlinear example, the superiority of ORDI is shown. It can be seen that the control law obtained by ORDI is much more concise than that of DEDI. Compared to DEDI, ORDI greatly simplifies the design process and reduce the computational burden. Furthermore, the performance of these two methods will be compared in the simulation.

C. Simulation

In the simulation, the initial values of system states are set to the equilibrium $[15060,110000,0,0.0312,0]^T$. The commands are set to let the velocity and altitude climb by 100 ft and 100 ft/s, respectively. The control parameters of DEDI are set as $k_{a11}=3$, $k_{a12}=3$, $k_{a13}=1$, $k_{a21}=4$, $k_{a22}=6$, $k_{a23}=4$, and $k_{a24}=1$. The control parameters of ORDI are set as $k_{c11}=1$, $k_{c21}=4$, and $k_{c22}=4$. Three values will be chosen for the combination coefficient λ to test its impact. Fig.5 shows the simulation results of DEDI and Figure 6 shows the simulation results of ORDI.

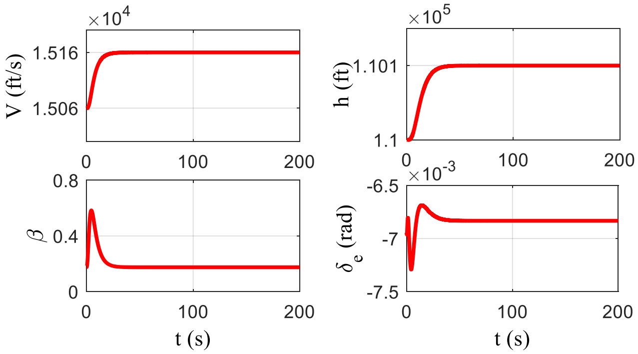


Figure 5. Simulation results of DEDI on hypersonic vehicle

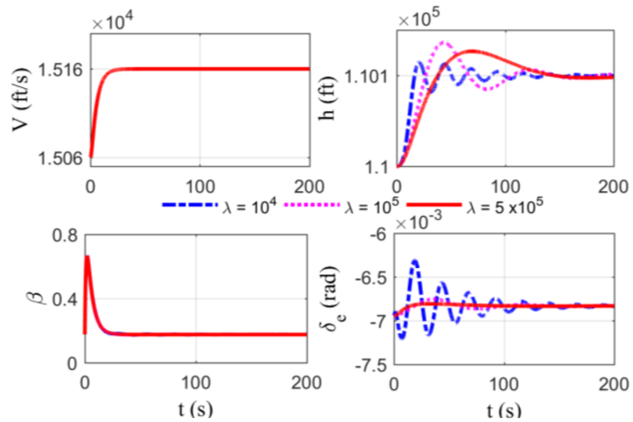


Figure 6. Simulation results of ORDI on hypersonic vehicle

For DEDI, as shown in Figure 5, the tracking performance is excellent that both outputs achieves the commands quickly and smoothly.

For ORDI, it is found that the altitude exhibits high-frequency vibration for small value of λ. This may because the root of the linearized zero dynamics has an imaginary part much greater than its real part when λ is small as shown in Figure 4. Therefore λ should be chosen as big numbers. The results are shown in Figure 6. It can be seen that the tracking performance of velocity is as good as that of DEDI, but the altitude response exhibits damped vibration. As λ increases, the vibration is weakened but the rising time become longer. So even though the altitude tracking performance can be adjusted by tuning the combination coefficient, the extent is limited by the structure of the nonlinear zero dynamics.

V. Conclusion

In this paper, we show that the output redefinition-based dynamic inversion (ORDI) method, which is originally proposed for nonminimum phase systems, is also applicable to control MIMO systems with no relative degree, thus enlarges the application range of ORDI. Compared to the traditional dynamic extension-based dynamic inversion (DEDI) method, ORDI gives a new option to the control issue for MIMO systems with no relative degree and has some advantages when applied to complex systems, which are shown in the hypersonic flight vehicle example. In a word, ORDI is much easier to implement and results in simpler control law thus reducing the computational burden. In the future, more work will be done to compare the robustness of these two methods when there are model uncertainties, and the new output will be chosen as a nonlinear combination to possibly improve the tracking performance.

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